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5.1 Notes – Bisectors of Triangles **5.2 Notes – Medians and Altitudes of Triangles**

G-CO.C.10 – Prove theorems about triangles; G-MG.A.2 – Apply geometric methods to solve real world problems

ESSENTIAL QUESTIONS: How can you use perpendicular bisectors to find the point that is equidistant from all of the vertices of a triangle? How can you use angle bisectors to find the point that is equidistant from the sides of the triangle?

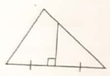
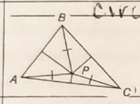
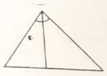
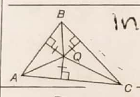
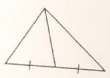
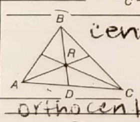
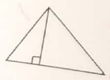
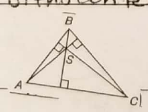
A segment, line, or plane that intersects a segment at its midpoint and is perpendicular to the segment is called a perpendicular bisector

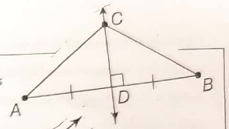
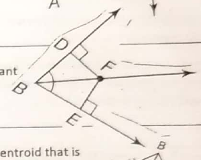
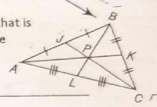
An angle bisector divides an angle into two congruent angles.

A median of a triangle is a segment with endpoints being a vertex of a triangle and the midpoint of the opposite side.

An altitude of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side.

The point where 3 or more lines intersect is called the point of concurrency

Name	Example	Point of Concurrency
PERPENDICULAR BISECTOR •forms right angle and splits the side into 2 congruent parts		 circumcenter
ANGLE BISECTOR •splits the angle into two congruent angles		 incenter
MEDIAN •splits the side into two congruent parts		 centroid
ALTITUDE •forms right angle		orthocenter 

PERPENDICULAR BISECTOR THEOREM If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment	
ANGLE BISECTOR THEOREM If a point is on the bisector of an angle, then it is equidistant from the sides of an angle.	
CENTROID THEOREM The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.	

5.3 Notes – Inequalities in One Triangle

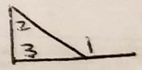
G.CO.C.10 – Prove theorems about theorems

G-MG.A.2 – Apply geometric methods to solve real-world problems

ESSENTIAL QUESTIONS: How can you use inequalities to describe the relationships among side lengths and angle measures in a triangle?

OBJECTIVES: Students will recognize and apply properties of inequalities to the measures of the angles of a triangle; Students will recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

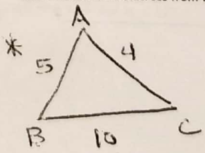
•REMINDER: The 2 remote interior angles add to the ext. angle.

Theorem 5.8: EXTERIOR ANGLE INEQUALITY
The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. $m\angle 1 > m\angle 2$ $m\angle 1 > m\angle 3$ 

ANGLE-SIDE INEQUALITIES

•The largest angle is across from the longest side

•The shortest side is across from the smallest angle



∠A is the largest angle (across from longest side)

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5.5 Notes – The Triangle Inequality

G-CO-C.10 - Prove theorems about triangles

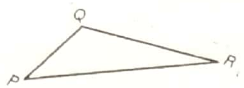
G-MG.A.2 – Apply geometry geometric methods to solve real-world problems

ESSENTIAL QUESTIONS: How can you use inequalities to describe the relationships among side lengths and angle measures in a triangle?

OBJECTIVE: Students will recognize and apply properties of inequalities to the measures of the angles of a triangle; Students will recognize and apply properties of inequalities to the relationships between the angles and sides of a triangle.

Theorem 5-11: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any sides of a triangle must be greater than the length of the third side.



PQ + QR > PR

QR + PR > PQ

PR + PQ > QR

5.6 Notes – Inequalities in Two triangles

G-CO.C.10 – Prove theorems about triangles

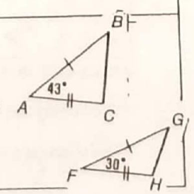
G-MG.A.2 – Apply geometric methods to solve real-world problems

ESSENTIAL QUESTIONS: In what ways can congruence be useful?

OBJECTIVES: Students will apply the Hinge Theorem or its converse to make comparisons in to triangles; Prove triangle relationships using the Hinge Theorem or its converse.

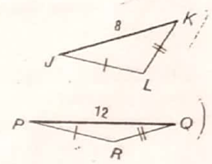
Theorems:

5.13: HINGE THEOREM: If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle



EX:

5.14: CONVERSE OF THE HINGE THEOREM: If two sides of a triangle are congruent to two sides of another triangle, and the third side in the first is longer than the third side in the second triangle, then the included angle measure of the first triangle is greater than the included angle measure in the second triangle



EX:

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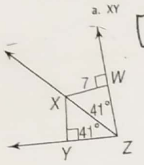
EXAMPLES FOR CHAPTER 5

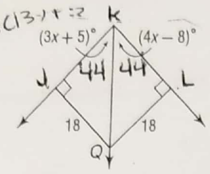
1. Find each measure

a. XY



$\boxed{XY=7}$

b. $m\angle JKL$



3(13)+5

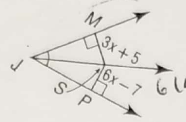
44 44

$3x+5=4x-8$

$13=x$

$\boxed{m\angle JKL=88}$

c. SP



6(4)-7

$6x-7=3x+5$

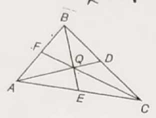
$3x=12$

$x=4$

$\boxed{SP=17}$

2. In ΔABC, Q is the centroid and BE = 9, QF = 6, and AQ = 14.

Find each measure.



a. BQ and QE

BQ = 6

QE = 3

b. FC and QC

QC = 12

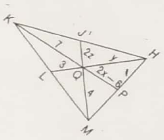
FC = 18

c. AD and QD

QD = 7

AD = 21

3. ALGEBRA. In the figure, if J, P, and L are midpoints of KH, HM, and MK, respectively, find x, y, and z



$2x-6=3.5$

$2x=9.5$

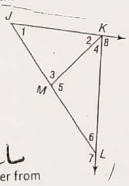
$\boxed{x=4.75}$

$2z=2$

$\boxed{z=1}$

$\boxed{y=6}$

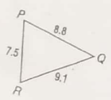
4. Use the Exterior Angle Inequality Theorem to list all of the angles that measure less than $m\angle 7$.



∠4, ∠5 for ΔKML

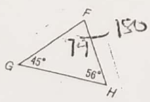
∠1, ∠2, ∠4 for ΔJKL

5. List the angles of ΔPQR in order from largest to smallest.



∠P, ∠R, ∠Q

6. List the sides of ΔFGH in order from shortest to longest



180 - 45 - 56

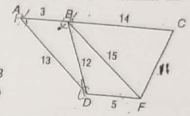
79

$\overline{FH}$, $\overline{FG}$, $\overline{GH}$

7. Use the figure to determine the relationship between the measures of the given angles

across from 13

across from 3



a. $m\angle ABD$, $m\angle BDA$

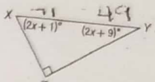
>

b. $m\angle BCF$, $m\angle CFB$

15 14

>

8. List the angles and sides of the triangle in order from smallest to largest.



71 49

$2x+1+2x+9+90=180$

$4x+100=180$

$4x=80$

$x=20$

angles

∠X, ∠Y, ∠Z

sides

$\overline{ZY}$, $\overline{XZ}$, $\overline{XY}$

9. Is it possible to form a triangle with the given side lengths? If not, explain why not.

a. 8 m., 15 m., 17 m.

$8+15>17$ ✓

$8+17>15$ ✓

$17+15>8$

yes

b. 6 cm, 8 cm, 14 cm

no $6+8 \not> 14$

10. Find the range for the measure of the third side of a triangle given the measures of two sides.

4 ft and 8 ft, x 5t

$4+x>8$

$x>4$

$8+x>4$

$-8 \quad -8$

$x>-4$

$4+8>x$

$12>x$

$\boxed{4<x<12}$

11. Find the range of possible measures of x if each set of expressions represents measures of the sides of triangle. $x, 2x+1, x+4$

$x+2x+1>x+4$

$3x+1>x+4$

$2x>3$

$\boxed{x>\frac{3}{2}}$

$x+x+4>2x+1$

$2x+4>2x+1$

$4>1$

$2x+1+x+4>x$

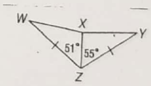
$3x+5>x$

$5>-2x$

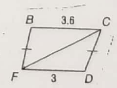
$-\frac{5}{2}<x$

Compare the given measures.

12. WX and XY

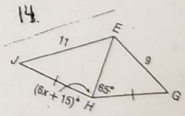


13. $m\angle FCD$ and $m\angle BFC$

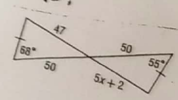


Find the range of possible values for x.

14.



15.



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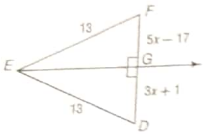
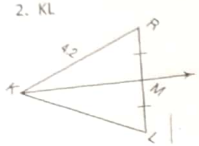
5.1 ASSIGNMENT

Find each measure.

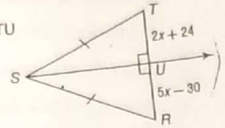
1./ FG



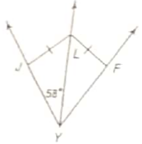
2. KL



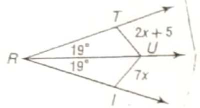
3. TU



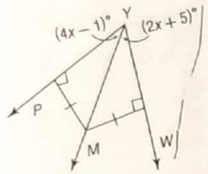
4. $m < LYF$



5. IU

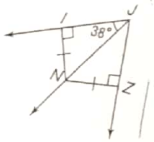


6. $m < MYW$

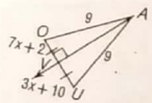


7. TP

8. $m < NJZ$



9. VU



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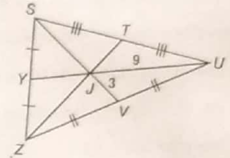
5.2 ASSIGNMENT

1./ In ΔSZU, UJ = 9, VJ = 3, and ZT = 18. Find each length.

a. YJ b. YU c. JT

d. SJ e. SV f. ZJ



2. Draw a triangle ΔRST with medians RM, SL, and TK, and centroid J

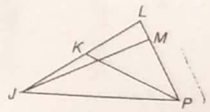
Complete each statement by finding x

a./ $SL = x\,(JL)$ b. $JT = x\,(TK)$ c. $JM = x\,(RJ)$

3. ALGEBRA. In ΔJLP, $m < JMP = 3x - 6$, $JK = 3y - 2$, $and\ LK = 5y - 8$

a. If $\overline{JM}$ is an altitude of ΔJLP, find x

b. Find LK if $\overline{PK}$ is a median.



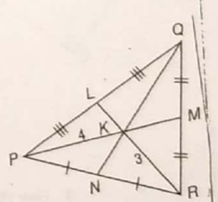
4. In ΔPQR, NQ = 6, RK = 3, and PK = 4.

Find each measure.

a./ KM b. KQ

c./ LK d. LR

e./ NK f. PM

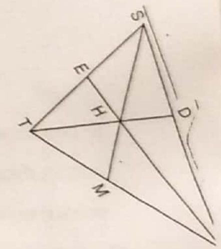


5. In ΔSTR, H is the centroid, EH = 6, DH = 4 and SM = 24. Find each measure

a. SH b. HM

c. TH d. LR

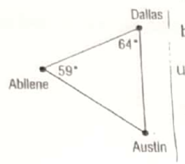
e. TD f. ER



5.3 ASSIGNMENT Name: Binder Copy

1. OBTUSE TRIANGLES. Don notices that the side opposite a right angle in a right triangle is always the longest side of the three sides. Is this also true of the side opposite the obtuse angle in an obtuse triangle? EXPLAIN

2. CITIES. Stella is going to Texas to visit a friend. As she was looking at a map to see where she might want to go, she noticed the cities Austin, Dallas, and Abilene formed a triangle. She wanted to determine how the distances between the cities were related, so she used a protractor to measure two angles

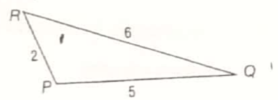


a. Based on the information in the figure, which of the two cities are nearest to each other?

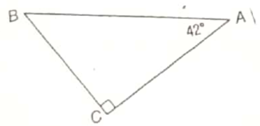
b. Based on the information in the figure, which of the two cities are farthest apart from each other?

List the angles AND sides of each triangle in order from smallest to largest.

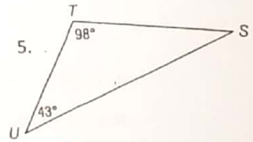
3.



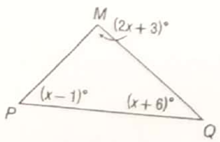
4.



5.



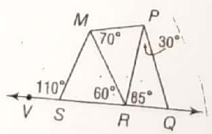
6. Find the value of x, measure of each angle, AND list the angles and sides in order from smallest to largest.



7. Use the figure to determine the relationship between the lengths of the given sides.

a. SM, MR

b. RP, MP



5.5 ASSIGNMENT Name: Binder Copy

1. CITIES. The distance between New York City and Boston is 187 miles and the distance between New York City and Hartford is 97 miles. Hartford, Boston, and New York City form a triangle on a map. What must the distance between Boston and Hartford be greater than?

2. STICKS. Jamila has 5 sticks of lengths 2, 4, 6, 8, and 10 inches. Using three sticks at a time as the sides of the triangles, how many triangles can she make?

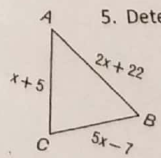
3. Is it possible to form a triangle with the given side lengths? If not explain why not and show work!

a. 9, 12, 18   b. 8, 9, 17   c. 14, 14, 19

4. Find the range for the measure of the third side of a triangle given the measures of two sides.

a. $6\ ft$ and $19\ ft$ b. $25\ yd$ and $38\ yd$ c. $54\ in$ and $7\ in$

5. Determine the possible values of x.



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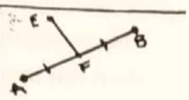
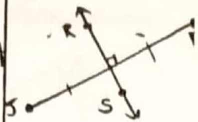
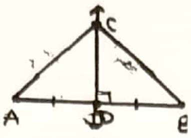
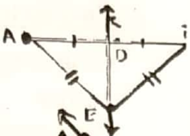
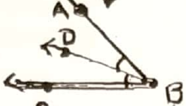
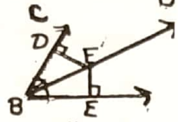
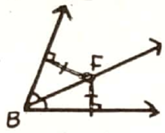
Bisectors of Triangles

Segment Bisector	any line segment or plane that intersects a segment at its midpoint	
Perpendicular Bisector	a bisector that is also perpendicular to the segment forms 4 right angles	
Perpendicular Bisector Theorem	If a point is on the ⊥ bisector of a segment, then it is equidistant from the endpoints of the segment	 AC = BC
Converse of ⊥ Bisector Theorem	If a point is equidistant from the endpoints of a segment, then it is on the ⊥ bisector of the segment	
Angle Bisector	any line, segment or ray that divides an angle into two congruent angles *	
Angle Bisector Theorem	If a point is on the bisector of an angle then it is equidistant from the sides of the angle	
Converse of Angle Bisector Theorem	If a point in the interior of an angle is equidistant from the sides of an angle, then it is on the bisector of the angle	
Incenter	The point of concurrency ...	Recall a Δ has * 3 angles so A... can have 3 angle ...

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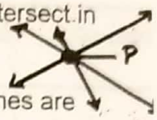
Objective: **Concurrent Lines**

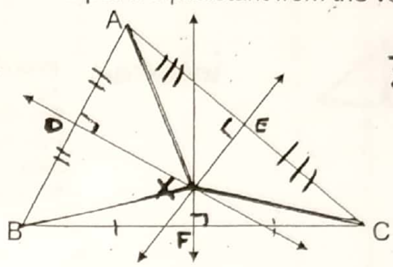
Concurrent – When three (3) or more lines intersect in one point.



Point of Concurrency – The point at which lines are concurrent.

Circumcenter – The point of concurrency of the perpendicular bisectors of a Δ

Theorem 4-16 Circumcenter Theorem
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.



If X is the circumcenter of triangle ABC (ΔABC) then

XA = XB = XC

Theorem 4-17
The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

The points of concurrency in Theorems 4-16 and 4-17 have some interesting properties related to circles.

Note: The circumcenter can be on the inside, out or a side of the triangle.

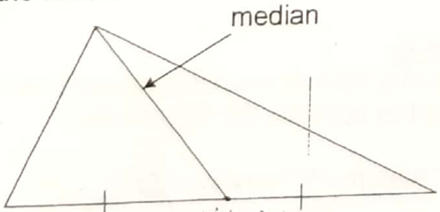
You can *circumscribe* circles around triangles. The *circumcenter* of the circle is equidistant from the vertices of the triangle.

You can *inscribe* a circle in a triangle. The *incenter* of the circle is equidistant from the sides of the triangle.

MEDIANS & ALTITUDES

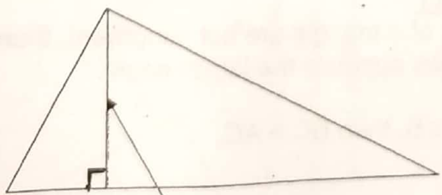
Median of a triangle – A segment of a triangle whose endpoints are a vertex and the midpoint of the side opposite the vertex.

median



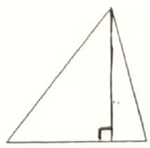
midpoint

Altitude of a triangle – A perpendicular segment from a vertex to the line containing the side opposite the vertex.

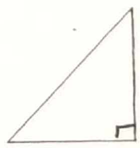


altitude

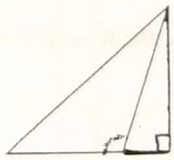
Unlike angle bisectors and medians, an altitude of a triangle may lie outside the triangle. Consider the following:



Acute Triangles: Altitude is inside.



Right Triangles: Altitude is a side.



Obtuse Triangles: Altitude is outside.

You can use paper folding to find altitudes and medians.

- To find an altitude – Fold so that a side overlaps itself and the fold contains a vertex.
- To find a median – Fold one vertex to another to find the midpoint of the side, then fold from the midpoint to the opposite vertex.

Theorem 4-18
The lines that contain the altitudes of a triangle are concurrent. The point of concurrency is called the orthocenter.

Theorem 4-19
The medians of a triangle are concurrent. The point of concurrency is called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

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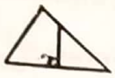
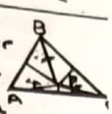
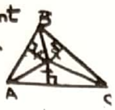
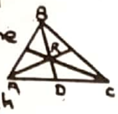
ummary: Special Segments and Points in Triangles

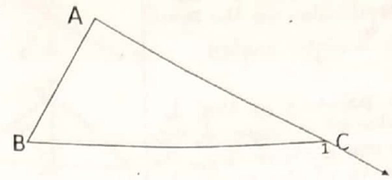
Name	Example	Point of Concurrency	Special Property	Example
rpendicular bisector		Circumcenter	Circumcenter is equidistant from each vertex	
ngle bisector		incenter	incenter is equidistant from each side of triangle	
edian		Centroid	centroid is $\frac{2}{3}$ the distance from each vertex to the midpoint of the opposite side	
ltitude		Orthocenter	lines containing altitudes are concurrent at the	

If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposites the lesser angle.

$m\angle J > m\angle K$ so $KL > JL$

Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.



$m\angle 1 > m\angle A$

$m\angle 1 > m\angle B$

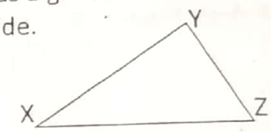
Angle-Side Inequalities

- The longest side and largest angle are opposite each other.
- The shortest side and the smallest angle are opposite each other.

Angle-Side Relationships in Triangles

If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

XY >YZ so $m\angle Z > m\angle X$

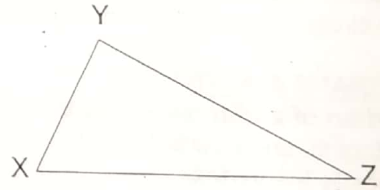


Objective: Triangle Inequalities

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

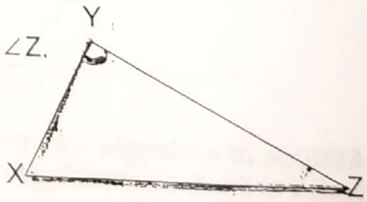
XY + YZ > XZ
YZ + XZ > XY
XZ + XY > YZ



Theorem 4-10

If two sides of a triangle are not congruent, then the larger angle lies opposite the larger side.

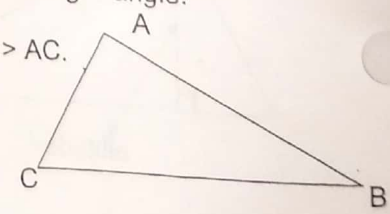
If XZ > XY, then m∠Y > m∠Z.



Theorem 4-11

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

If m∠A > m∠B, then BC > AC.



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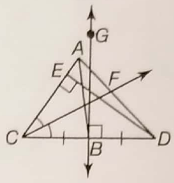
Chapter 5 Challenge Questions

Name: Binder Copy

Tier 1

Use the diagram for questions #1 – 3, to determine which is a true statement for the given information



1. $\overline{AC}$ is a median

a. $m < ACD = 90$ b. $< BAC \cong < DAC$ c. $BC = CD$ d. $< B \cong < D$

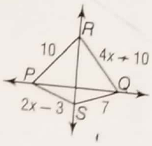
2. $\overline{AC}$ is an angle bisector.

a. $m < ACD = 90$ b. $< BAC \cong < DAC$ c. $BC = CD$ d. $< B \cong < D$

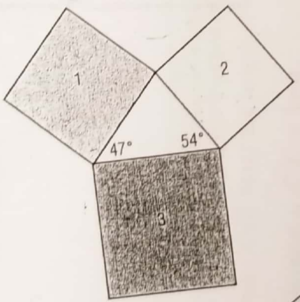
3. $\overline{AC}$ is an altitude.

a. $m < ACD = 90$ b. $< BAC \cong < DAC$ c. $BC = CD$ d. $< B \cong < D$

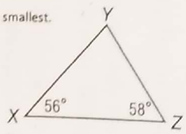
4. The perimeter of PRQS is 34. Find the value of x. Then describe the relationship between $\overline{RS}$ and $\overline{PQ}$



5. SQUARES. Matthew has three different squares. He arranges the squares to form a triangle as shown. Based on the information shown, list the squares in order from the one with the smallest perimeter to the one with the largest perimeter.



6. List the sides AND angles in order from largest to smallest.

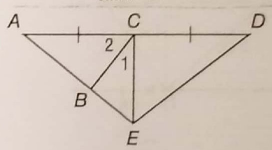


Tier 2

1. What is the range of lengths of each leg of an isosceles triangle if the measure of the base is 6 inches? EXPLAIN

2. WRITING IN MATH. Explain why the hypotenuse of a right triangle is always the longest side of the triangle.

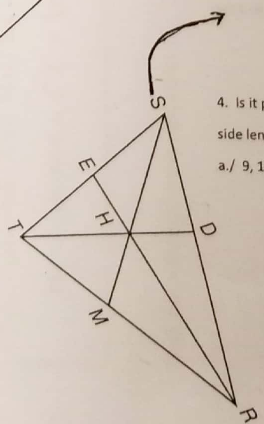
3. If $\overline{EC}$ is an altitude of ΔAED, $m < 1 = 2x + 7$, and $m < 2 = 3x + 13$, find m<1 and m<2



4. Find the longest side of ΔABC if $m < A = 70$, $m < B = 2x - 10$, and $m < C = 3x + 20$

3. In ΔSTR, H is the centroid, EH = 8, DH = 4 and SM = 36. Find each measure

a. SH b. HM c. TH d. HR e. TD f. ER



4. Is it possible to form a triangle with the given side lengths? EXPLAIN

a. 9, 12, 18 b. 23, 26, 50

5. Find the range of possible measures of x if each set of expressions represents the sides of a triangle.

$x + 2, x + 4, x + 6$

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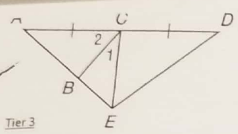


Tier 3

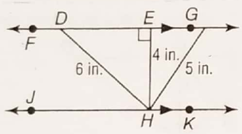
1. a. Find the value of x if $AC = 4x - 3$, $DC = 2x + 9$, $m < ECA = 15x + 2$ and $\overline{EC}$ is a median of ΔAED.

b. Is $\overline{EC}$ also an altitude of ΔAED? EXPLAIN.

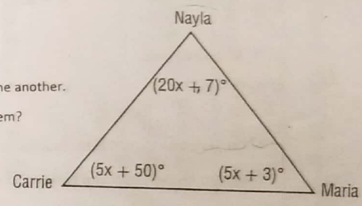
6. $\overline{QS}$ is a median of ΔPQR with point S on $\overline{PR}$. If $PS = x^2 - 3x$ and $SR = 2x + 6$, find the possible value(s) of x.

2. CITIES. The distance between New York City and Boston is 187 miles and the distance between New York City and Hartford is 97 miles. Hartford, Boston, and New York City form a triangle on a map. What must the distance between Boston and Hartford be greater than?

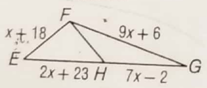
3. Mary says that $\overline{HE}$ is a perpendicular bisector of $\overline{DG}$ and Ashley says it is not. Who is correct? EXPLAIN



7. Carrie, Maria, and Nayla are friends that live close to one another. Which two friends have the shortest distance between them?

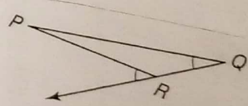


4. If $\overline{FH}$ is a median of ΔEFG, find the perimeter of ΔEFG.



5. $\overline{HJ}$ is an altitude of ΔGHI with point J on $\overline{GI}$. If $m < GJH = 5x + 30$, $GH = 3x + 4$, $HI = 5x - 3$, $JI = 4x - 3$, and $GJ = x + 6$, find the perimeter of ΔGHI

8. WRITING IN MATH. Analyze the information given in the diagram and explain why the markings must be incorrect



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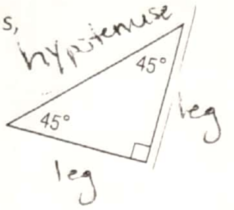
8.3 Notes (PART 1)– Special Right Triangles

G-SRT.C.8 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

ESSENTIAL QUESTION: How do you find side length or angle measures in a right triangle? How do trigonometric ratios relate to similar right triangles?

OBJECTIVES: Identify and apply side ratios in 45-45-90 triangles; Identify and apply side ratios in 30-60-90 triangles.

*In a 45° – 45° – 90° triangles,



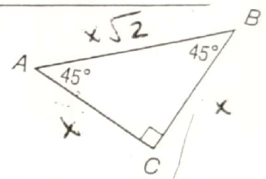
* since the angles are congruent, both 45°, the legs are CONGRUENT!

Theorem 8.8: 45-45-90 Triangle Theorem

In a 45° – 45° – 90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg

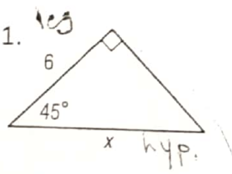
hypotenuse = $\sqrt{2} \cdot$ leg

*



Find the value of each variables.

1.

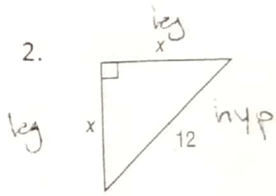


hyp = $\sqrt{2} \cdot$ leg

$x = \sqrt{2} \cdot 6$

$\boxed{x = 6\sqrt{2}}$

2.



*since both legs are x, the angle are equa So 45°

hyp = $\sqrt{2} \cdot$ leg

$12 = \sqrt{2}\,x$

$\boxed{8.5 = x}$ ← divide by $\sqrt{2}$.

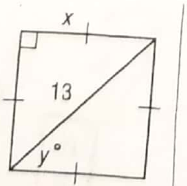
or $6\sqrt{2}$

Find x and y.

legs are equal, so both angles are 45°, so

$\boxed{y = 45°}$

3.

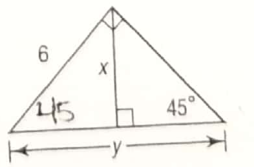


hyp = $\sqrt{2} \cdot$ leg

$13 = \sqrt{2}\,x$ or $\frac{13\sqrt{2}}{2}$

$\boxed{9.2 = x}$ ← divide by $\sqrt{2}$

4.



hyp = $\sqrt{2} \cdot$ leg

$6 = \sqrt{2} \cdot x$

$\boxed{\begin{array}{l} 4.2 = x \\ 8.4 = y \end{array}}$ or $x = 3\sqrt{2}$

$y = 6\sqrt{2}$

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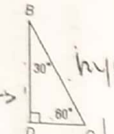
8.3 Notes (PART 1)– Special Right Triangles

G-SRT.C.8 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

<u>ESSENTIAL QUESTION:</u> How do you find side length or angle measures in a right triangle? How do trigonometric ratios relate to similar right triangles?

<u>OBJECTIVES:</u> Identify and apply side ratios in 45-45-90 triangles; Identify and apply side ratios in 30-60-90 triangles.

•In a $30° - 60° - 90°$ triangle,



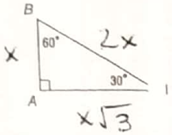
hypotenuse

long leg →

short leg

Theorem 8.9 30-60-90 triangle theorem

In a $30° - 60° - 90°$ triangle, the length of the hypotenuse is 2 times the length of the shorter leg s, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg.



x

2x

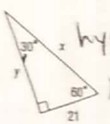
$x\sqrt{3}$

hypotenuse = 2 · short leg

long leg = $\sqrt{3}$ · short leg

Find the value of each variable.

1.



hyp

long

short

hyp = 2 · short

x = 2 · 21

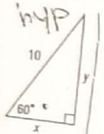
$\boxed{x = 42}$

long = $\sqrt{3}$ · short

$y = \sqrt{3} \cdot 21$

$\boxed{y = 21\sqrt{3}}$

2.



hyp

long

short

hyp = 2 · short

10 = 2x

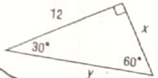
$\boxed{5 = x}$

long = $\sqrt{3}$ · short

$y = \sqrt{3} \cdot 5$

$\boxed{y = 5\sqrt{3}}$

3.



long

short

hyp

hyp = 2 · short

y = 2(6.9)

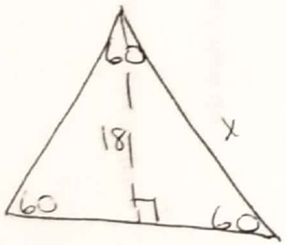
$\boxed{y = 13.9}$ or $8\sqrt{3}$

long = $\sqrt{3}$ · short

$12 = \sqrt{3}\, x$ ← divide it

$\boxed{6.9 = x}$ or $4\sqrt{3}$

4. An equilateral triangle has an altitude length of 18 feet. Determine the length of a side of the triangle.



long = $\sqrt{3}$ · short

$18 = \sqrt{3}$ short

10.4 = short or $6\sqrt{3}$

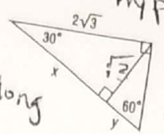
hyp = 2 · short

x = 2(10.4)

$\boxed{x = 20.8 \text{ ft}}$ or $12\sqrt{3}$

Find each variable.

5.



hyp

long

hyp = 2 · short

$2\sqrt{3} = 2$ short

$\sqrt{3}$ = short

long = $\sqrt{3}$ · short

long = $\sqrt{3} \cdot \sqrt{3}$

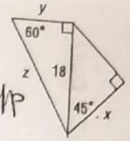
$\boxed{x = 3}$

long = $\sqrt{3}$ · short

$\sqrt{3} = \sqrt{3}\, y$ ← divide

$\boxed{1 = y}$

6.



short

hyp

hyp = $\sqrt{2}$ · leg

$18 = \sqrt{2}\, x$

$\boxed{12.7 = x}$

long = $\sqrt{3}$ · short

$18 = \sqrt{3}\, y$

$\boxed{10.4 = y}$ or $6\sqrt{3}$

hyp = 2 · short

z = 2(10.4)

$\boxed{z = 20.8}$

or $12\sqrt{3}$

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5.3 Notes (PART 1)– Special Right Triangles

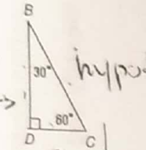
G-SRT.C.6 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

ESSENTIAL QUESTION: How do you find side length or angle measures in a right triangle? How do trigonometric ratios relate to similar right triangles?

OBJECTIVES: Identify and apply side ratios in 45-45-90 triangles; Identify and apply side ratios in 30-60-90 triangles.

•In a 30° – 60° – 90° triangle,

long leg →

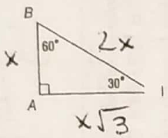


hypotenuse

short leg

Theorem 8.9 30 - 60 - 90 triangle theorem

In a 30° – 60° – 90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg s, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg.



x

2x

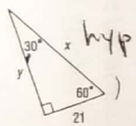
$x\sqrt{3}$

hypotenuse = 2 · short leg

long leg = $\sqrt{3}$ · short leg

Find the value of each variable.

1.



hyp

long

short

hyp = 2 · short

x = 2 · 21

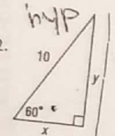
$\boxed{x = 42}$

long = $\sqrt{3}$ · short

y = $\sqrt{3}$ · 21

$\boxed{y = 21\sqrt{3}}$

2.



hyp

long

short

hyp = 2 · short

10 = 2x

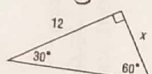
$\boxed{5 = x}$

long = $\sqrt{3}$ · short

y = $\sqrt{3}$ · 5

$\boxed{y = 5\sqrt{3}}$

3.



long

short

hyp

hyp = 2 · short

y = 2(6.9)

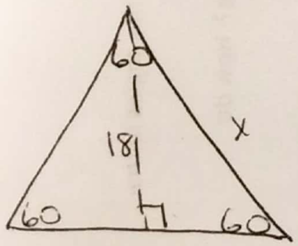
$\boxed{y = 13.9}$ or $8\sqrt{3}$

long = $\sqrt{3}$ · short

12 = $\sqrt{3}$ x ← divide it

$\boxed{6.9 = x}$ or $4\sqrt{3}$

4. An equilateral triangle has an altitude length of 18 feet. Determine the length of a side of the triangle.



long = $\sqrt{3}$ · short

18 = $\sqrt{3}$ short

10.4 = short or $6\sqrt{3}$

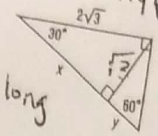
hyp = 2 · short

x = 2(10.4)

$\boxed{x = 20.8 \text{ ft}}$ or $12\sqrt{3}$

Find each variable.

5.



hyp

long

hyp = 2 · short

$2\sqrt{3}$ = 2 short

$\sqrt{3}$ = short

long = $\sqrt{3}$ · short

long = $\sqrt{3}$ · $\sqrt{3}$

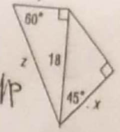
$\boxed{x = 3}$

long = $\sqrt{3}$ · short

$\sqrt{3}$ = $\sqrt{3}$ y ← divide

$\boxed{1 = y}$

6.



short

hyp

hyp = $\sqrt{2}$ · leg

18 = $\sqrt{2}$ x

$\boxed{12.7 = x}$

long = $\sqrt{3}$ · short

18 = $\sqrt{3}$ y

$\boxed{10.4 = y}$ or $6\sqrt{3}$

hyp = 2 · short

z = 2(10.4)

$\boxed{z = 20.8}$ or $12\sqrt{3}$

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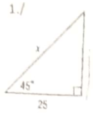
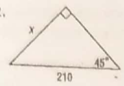
8.3 ASSIGNMENT (PART 1) Name: Binder Copy

Find the value of each variable.

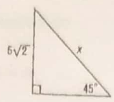
1.



2.



3.

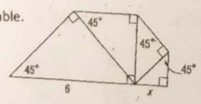


4. ORIGAMI. A square piece of paper 150 mm on a side is folded in half along a diagonal. The result is a $45° - 45° - 90°$ triangle. What is the length of the hypotenuse of the triangle?

5. Determine the length of the leg of a $45° - 45° - 90°$ triangle with a hypotenuse length of 26.

6. Find the length of the hypotenuse of a $45° - 45° - 90°$ triangle with a leg length of 50 cm.

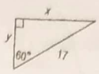
7. Find each variable.



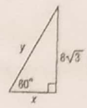
8.3 ASSIGNMENT (PART 2) Name: Binder Copy

Find the value of each variable.

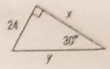
1.



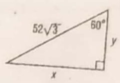
2.



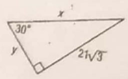
3.



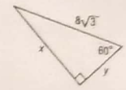
4.



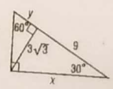
5.



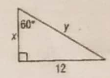
6.

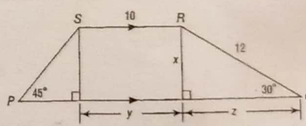


7.



8





9. **HONORS only, but others can get extra credit** Find x, y, and z, and the perimeter of trapezoid PQRS.

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* Make sure calculator is in DEGREE mode...

8.4 Notes (PART 1)– Trigonometry

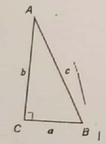
G-SRT.C.8 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

ESSENTIAL QUESTION: How do you find side length or angle measures in a right triangle? How do trigonometric ratios relate to similar right triangles?

OBJECTIVES: Define trigonometric ratios for acute angles in right triangles; Use the relationship between the sines and cosines of complementary angles.

A Trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

If ΔABC is a right triangle with acute <A, then the Sine of <A (written as Sin A) is the ratio of the length of the leg opposite <A to the length of the hypotenuse.	$\sin A = \frac{opp}{hyp} = \frac{a}{c}$ $\sin B = \frac{b}{c}$
If ΔABC is a right triangle with acute <A, then the cosine of <A (written as Cos A) is the ratio of the length of the leg adjacent to <A to the length of the hypotenuse.	$\cos A = \frac{adj}{hyp} = \frac{b}{c}$ $\cos B = \frac{a}{c}$
If ΔABC is a right triangle with acute <A, then the tangent of <A (written as tan A) is the length of the leg opposite <A to the length of the leg adjacent to <A	$\tan A = \frac{opp}{adj} = \frac{a}{b}$ $\tan B = \frac{b}{a}$



*To help remember the formulas:

SOHCAHTOA

sin $\frac{opp}{hyp}$ cos $\frac{adj}{hyp}$ tan $\frac{opp}{adj}$

*To set up problems:

angle

$\sin A = \frac{opp}{hyp}$ 2 sides

1./ Express each ratio as a fraction and as a decimal to the nearest hundredth.

a. Sin P

$\frac{opp}{hyp} = \frac{15}{17}$ or 0.88

b. Cos P

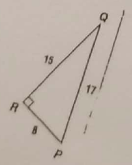
$\frac{adj}{hyp} = \frac{8}{17}$ or 0.47

c. Tan P $\frac{opp}{adj}$

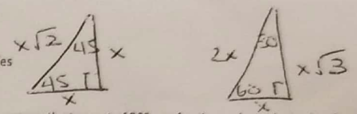
$\frac{15}{8}$ or 1.88

d. Sin Q

$\frac{opp}{hyp} = \frac{8}{17}$ or 0.53



*Reminder: Special right triangles



2. Use a special right triangle to express the tangent of 30° as a fraction and as a decimal to the nearest hundredth.

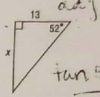
$\tan 30 = \frac{opp}{adj}$

$\tan 30 = \frac{x}{x\sqrt{3}}$

$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ or 0.58

Find the value of x to the nearest hundredth.

3.



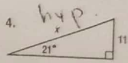
$\tan 52 = \frac{x}{13}$

put it over 1 and cross multiply

$x = 13 \tan 52$

$x = 16.6$

4.

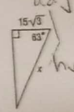


$\sin 21 = \frac{11}{x}$

$\frac{x \sin 21}{\sin 21} = \frac{11}{\sin 21}$

$x = 30.7$

5.



$\cos 63 = \frac{15\sqrt{3}}{x}$

$x \cos 63 = 15\sqrt{3}$

$x = \frac{15\sqrt{3}}{\cos 63}$

$x = 57.2$

6. HIKING. A certain part of a hiking trail slopes upward at about a 5° angle. After traveling a horizontal distance of 100 feet along this part of the trail, what would be the change in the hiker's vertical position? What distance has the hiker traveled along the path?

$\tan 5 = \frac{x}{100}$

$x = 100 \tan 5$

$x = 8.8$ ft

$\cos 5 = \frac{100}{y}$

$y \cos 5 = 100$

$y = \frac{100}{\cos 5}$

$y = 100.4$ ft

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8.4 Notes (PART 2)– Trigonometry

G-SRT.C.8 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

ESSENTIAL QUESTION: How do you find side length or angle measures in a right triangle? How do trigonometric ratios relate to similar right triangles?

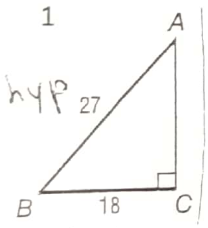
OBJECTIVES: Define trigonometric ratios for acute angles in right triangles; Use the relationship between the sines and cosines of complementary angles.

*Reminder: SOH CAH TOA

Key Concept: Inverse Trigonometric Ratios
*Use when: you are solving for angles
If <A is an acute angle and the sine of <A is x, then the inverse sine of x is the measure of <A.
Symbol: $\sin A = x$, then $A = \sin^{-1}(x)$
* hit $\boxed{\text{shift}}$ sin in calculator

Find the measure of <A to the nearest whole number.

1

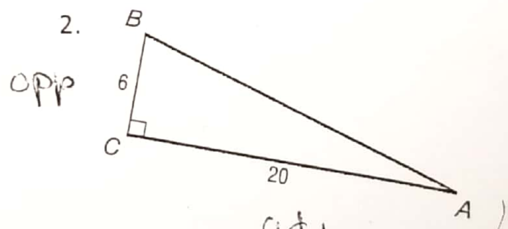


$\sin A = \frac{18}{27}$

$\sin^{-1}\left(\frac{18}{27}\right)$

$\angle A = 42^\circ$

2.

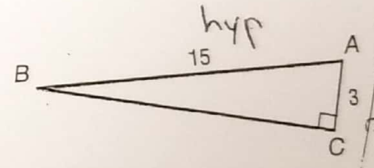


$\tan A = \frac{6}{20}$

$\tan^{-1}\left(\frac{6}{20}\right)$

$\angle A = 17^\circ$

3.



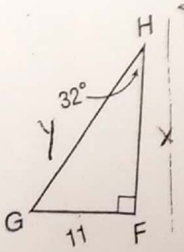
$\cos A = \frac{3}{15}$

$\cos^{-1}\left(\frac{3}{15}\right)$

$\angle A = 78^\circ$

4. Solve the triangle. Round sides to the nearest tenth and angles to the nearest degree.

*find all missing sides/angles



$\boxed{\angle G = 180 - 32 - 90 = 58}$

$\tan 32 = \frac{11}{x}$

$x \tan 32 = 11$

$x = \frac{11}{\tan 32}$

$\boxed{HF = 17.6}$

$\sin 32 = \frac{11}{y}$

$y \sin 32 = 11$

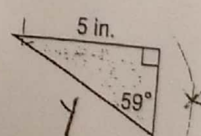
$y = \frac{11}{\sin 32}$

$\boxed{GH = 20.8}$

5. HONORS. Find the perimeter and area of the triangle.

add all sides

$\frac{1}{2}bh$



$\tan 59 = \frac{5}{x}$

$x \tan 59 = 5$

$x = \frac{5}{\tan 59}$

$\sin 59 = \frac{5}{y}$

$y \sin 59 = 5$

$y = \frac{5}{\sin 59}$

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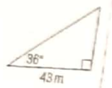
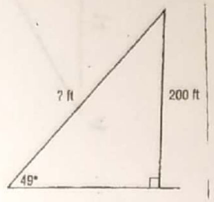
8 Trigo GNMENT (PART 1)

Name: Binder Copy

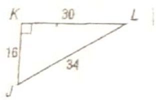
1. GEOGRAPHY. Diego used a theorolite to map a region of land for his class in geomorphology. To determine the angle of elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of site to the top of the formation. The distance was 43 m and the angle was 36°. What is the height of the formation to the nearest meter?



2. RADIO TOWERS. Kay is standing near a 200-foot-high radio tower. Use the information in the figure to determine how far Kay is from the top of the tower.

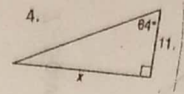


3. Find sin J, cos J, tan J, sin L, cos L, and tan L. Express each ratio as a fraction and as a decimal rounded to the nearest hundredth.

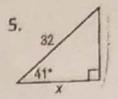


Find x. Round to the nearest hundredth

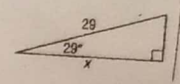
4.



5.



6



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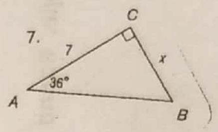

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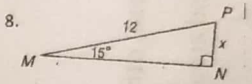

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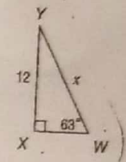


HONORS must complete these, extra points for others

Use a special right triangle to express each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth

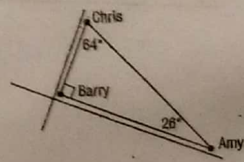
10. tan 60°

11. sin 45°

12. NEIGHBORS. Amy, Barry, and Chris live on the same block. Chris lives up the street and around the corner from Amy, and Barry lives at the corner between Amy and Chris. The three homes are the vertices of a right triangle.

a. Give two trigonometric expressions for the ratio of Barry's distance from Amy to Chris' distance from Amy.

b. Give two trigonometric expressions for the ratio of Barry's distance from Chris to Amy's distance from Chris.



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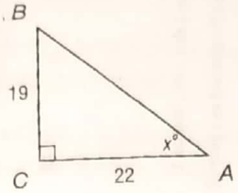
8.4 ASSIGNMENT (PART 2) Name: Binder Copy

Find the value of x.

1.

2.



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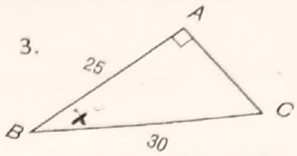
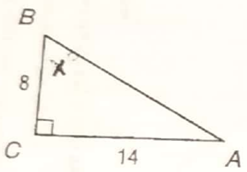


4.



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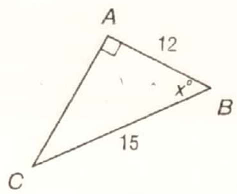
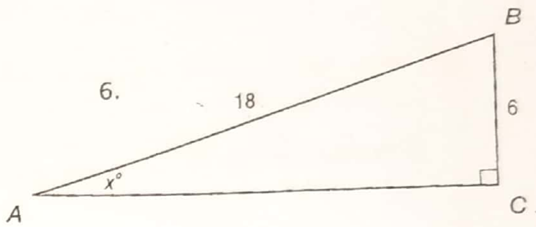
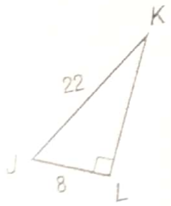


6.

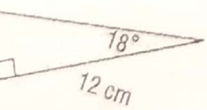


7. Solve the right triangle. Round sides to the nearest tenth and angles to the nearest degree.

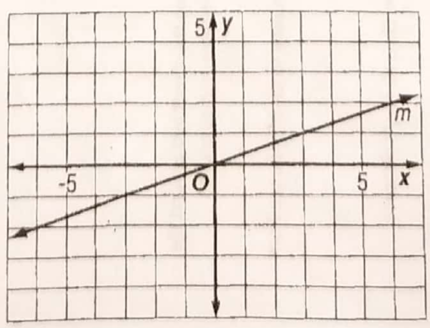


HONORS must solve these, others can get extra credit.

8. Find the perimeter and area



9. LINES. Jasmine draws line m on a coordinate plane. What angle does m make with the x-axis?



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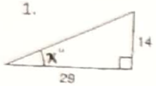
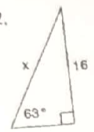
Trigonometry Review Name: Binder Copy

Find each variable. Round sides to the nearest tenth and angles to the nearest degree.

1.



2.



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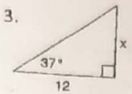

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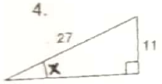
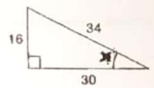


5.

6.



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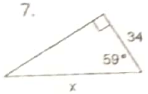
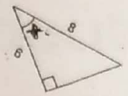


8.

9.

10.



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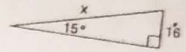

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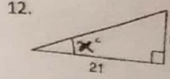

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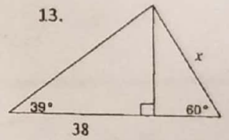

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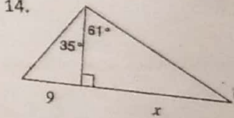
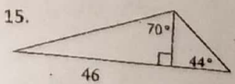


Find the area of each triangle. $A = \frac{1}{2}bh$

15.



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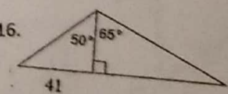
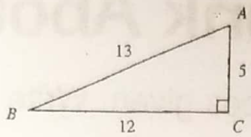


The Sine, Cosine, and Tangent Ratios

For use after Section 8–6

Use the diagram to express the ratio as a fraction.

1. sin A = ________ 2. cos A = ________
3. cos B = ________ 4. tan A = ________
5. tan B = ________ 6. sin B = ________



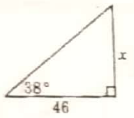
Exs. 1–6

Complete. Use a scientific calculator or the table on page 311 of the text.

7. sin 3° ≈ ________ 8. cos 30° ≈ ________
9. tan 48° ≈ ________ 10. sin 79° ≈ ________
11. cos ________ ≈ 0.9455 12. sin ________ ≈ 0.8746
13. tan ________ ≈ 2.4751 14. cos ________ ≈ 0.6428

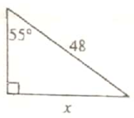
Use a scientific calculator or the table on page 311 of the text to find the values of the variables. Find lengths correct to the nearest integer and angles to the nearest degree.

15.



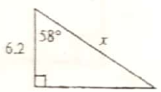
x ≈ ________

16.



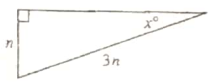
x ≈ ________

17.



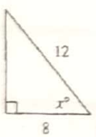
x ≈ ________

18.



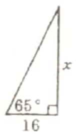
x ≈ ________

19.



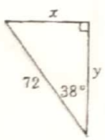
x ≈ ________

20.



x ≈ ________

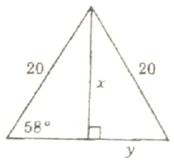
21.



x ≈ ________

y ≈ ________

22.



x ≈ ________

y ≈ ________

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Binder

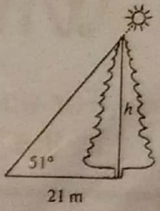
Applications of Trigonometry

For use after Section 8–7

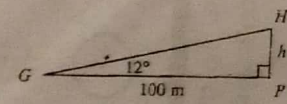
B,

In Exercises 1–5 express lengths correct to the nearest meter and angles correct to the nearest degree. Use a scientific calculator or the table on page 311 of the text.

1. A tree casts a shadow 21 m long. The angle of elevation of the sun is 51°. What is the height of the tree? ________

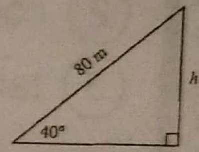


2. A helicopter (H) is hovering over a landing pad (P) 100 m from where you are standing (G). The helicopter's angle of elevation with the ground is 12°. What is the altitude of the helicopter? ________

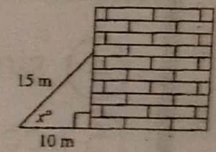


3. You are flying a kite and have let out 80 m of string. The kite's angle of elevation with the ground is 40°. If the string is stretched straight, how high is the kite above the ground?

____

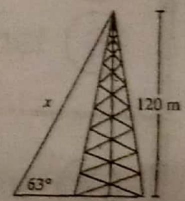


4. A 15 m pole is leaning against a wall. The foot of the pole is 10 m from the wall. Find the angle the pole makes with the ground. ________



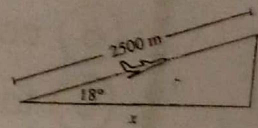
5. A guy wire reaches from the top of a 120 m television transmitter tower to the ground. The wire makes a 63° angle with the ground. Find the length of the guy wire.

____



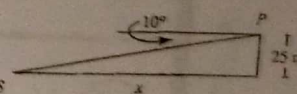
6. An airplane climbs at an angle of 18° with the ground. Find the ground distance the plane travels as it moves 2500 m through the air. Give your answer to the nearest 100 m.

____



7. A lighthouse operator at point P 25 m above sea level sights a sailboat at point S. The angle of depression of the sighting is 10°. How far is the boat from the base of the lighthouse? Give your answer to the nearest 10 m.

____



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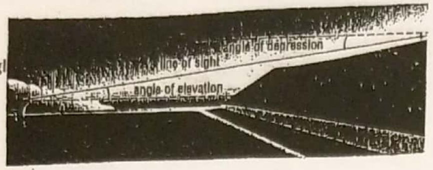
8.5 Notes – Angles of Elevation and Depression

G-SRT.C.8 - Solve triangles: a. Know and use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems

ESSENTIAL QUESTIONS: How do you find a side length or angle measure in a right triangle; How do trigonometric ratios relate to similar right triangles?

OBJECTIVES: Solve problems involving angles of elevation; Solve problems involving angles of depression

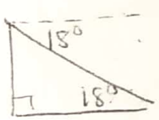


An angle of elevation is the angle formed by a horizontal line and an observer's line of sight to an object above the horizontal line.

An angle of depression is the angle formed by a horizontal line and an observer's line of sight to an object below the horizontal line.

*Horizontal Lines are parallel, so the angle of elevation and angle of depression are Alternate Interior (congruent

1./ INDIRECT MEASUREMENT. Miguel looks out from the crown of the statue of liberty approximately 250 feet above the ground. He sights a ship coming in New York Harbor and measures the angle of depression as 18°. Find the distance from the base of the statue to the ship to the nearest foot.

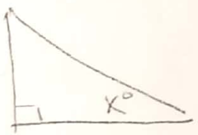


$\tan 18 = \frac{250}{x}$

$x = \frac{250}{\tan 18}$

$\boxed{x = 769 \text{ ft}}$

2. FLAGPOLE. The world's tallest unsupported flagpole is a 282-ft-tall steel pole in Surrey, British Columbia. The shortest shadow cast by the pole during the year is 137 feet long. To the nearest degree, what is the angle of elevation of the sun when the shortest shadow is cast?



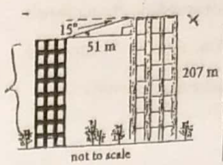
$\tan x = \frac{282}{137}$

$\tan^{-1}\left(\frac{282}{137}\right)$

$= 64°$

3. CONSTRUCTION. Two office buildings are 51 m apart. The height of the taller building is 207 m. The angle of depression from the top of the taller building to the top of the shorter building is 15°. Find the height of the shorter building to the nearest meter.

find height



$\tan 15 = \frac{x}{51}$

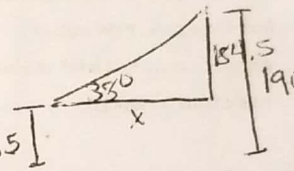
$x = 51 \tan 15$

$x = 14$

$207 - 14$

$\boxed{193 \text{ feet}}$

4. VACATION. Leah is meeting friends a the castle in the center of an amusement park. She sights the top of the castle at an angle of elevation of 38°. From the park's brochure, she knows that the castle is 190 feet tall. If Leah is 5.5 feet tall, about how far is she from the castle to the nearest foot?

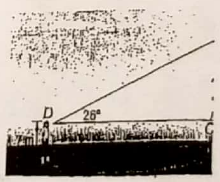


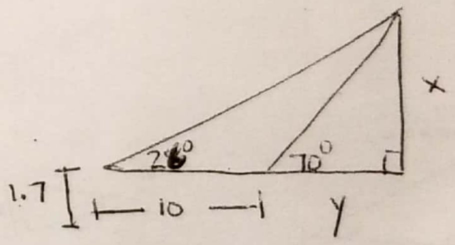
$\tan 38 = \frac{184.5}{x}$

$x = \frac{184.5}{\tan 38}$

$\boxed{236 \text{ feet}}$

5. **HONORS only.** TREE REMOVAL. To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?





$\tan 26 = \frac{x}{y+10}$

$\underbrace{(y+10)\cdot\tan 26}$ $= x$

$\tan 70 = \frac{x}{y}$

$\underbrace{y \tan 70 = x}$

$(y+10)\tan 26 = y \tan 70$

$y \tan 26 + 10 \tan 26 = y \tan 70$

$10 \tan 26 = y \tan 70 - y \tan 26$

$10 \tan 26 = y(\tan 70 - \tan 26)$

$2.16 = y$ ← plug it in to get x

$2.16 \tan 70 = x$

$5.9 = x$

height of tree: $5.9 + 1.7$

$= 7.6$ or 8 meters

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8.5 ASSIGNMENT

Name: Binder Copy

1. You sight a rock climber on a cliff at a 32° angle of elevation. The horizontal ground distance to the cliff is 1000 feet. Find the line-of-sight distance to the rock climber.

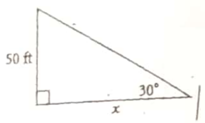
2. An airplane pilot sights a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's surface distance from the raft?

3. A 6 ft tall man stands 12 feet from the base of a tree. The angle of elevation from his eyes to the top of the tree, about 76°. How tall is the tree?

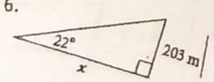
4. BASEBALL. A fan is seated in the upper deck of a stadium 200 feet away from home plate. If the angle of depression to the field is 62°, at what height is the fan sitting?

Find the value of x.

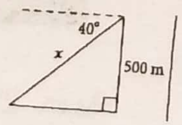
5.



6.

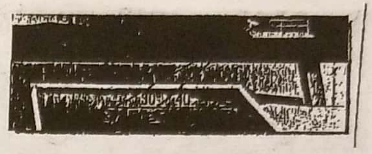


7.



HONORS must complete these, others can for extra points

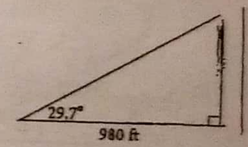
8. DIVING. Austin is standing on the high dive at the local pool. Two of his friends are in the water as shown. If the angle of depression to one of his friends is 40°, and 30° to his other friend who is 5 feet beyond the first, how tall is the platform?



9. HYDROMECHANICS. An engineer is 980 feet from the base of a fountain at Fountain Hills, Arizona. The angle of elevation to the top of the column of water is 29.7°. The surveyor's angle measuring device is at the same level as the base of the fountain.

a. Find the height of the column of water to the nearest 10 ft.

b. When the top of the column of water is just half as high as in part a, find the angle of elevation to its top.



10. ERROR ANALYSIS. Terrence and Rodrigo are trying to determine the relationship between the angles of elevation and depression. Terrence says that if you are looking up at someone with an angle of elevation of 35°, then they are looking down at you with an angle of depression of 55°, which is the complement of 35°. Rodrigo disagrees and says that the other person would be looking down at you with an angle of depression equal to your angle of elevation, or 35°. Is either of them correct? Explain.

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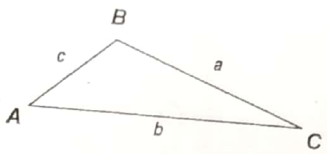
8.6 Notes – The Law of Sines and Law of Cosines

G-SRT.C.8 – Solve triangles. Know and use the Law of Sines and Law of Cosines to solve problems in real life situations. Recognize when it is appropriate to use each.

*The **LAW OF SINES** can be used for nonright triangles

THEOREM 8.10 LAW OF SINES

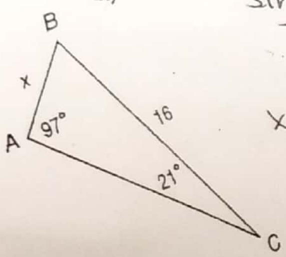
If ΔABC had lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Find x. Round to the nearest tenth.

1.

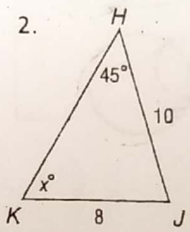


$$\frac{\sin 21}{x} = \frac{\sin 97}{16}$$

$$\frac{x \cancel{\sin 97}}{\cancel{\sin 97}} = \frac{16 \sin 21}{\sin 97}$$

$$\boxed{x = 5.8}$$

2.



$$\frac{\sin x}{10} = \frac{\sin 45}{8}$$

$$\frac{\cancel{8} \sin x}{\cancel{8}} = \frac{10 \sin 45}{8}$$

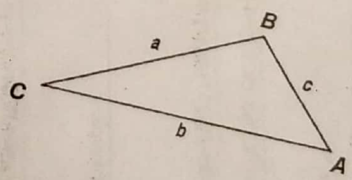
$$\sin x = \frac{5\sqrt{2}}{8}$$

$$\boxed{x = 62^\circ}$$

*You can use the **LAW OF COSINES** to solve a triangle if you know the measures of two sides and the included angle (SAS)

THEOREM 8.11 LAW OF COSINES

If ΔABC has lengths a, b, and c, representing the lengths of the sides opposite the angles with measures A, B, and C, then

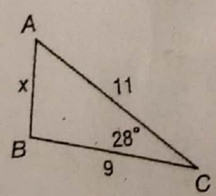


$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

3.



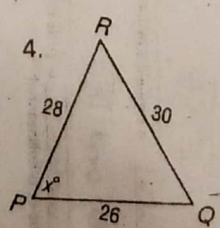
$$x^2 = 9^2 + 11^2 - 2(11)(9) \cos 28$$

$$x^2 = 27.2$$

$$x = \sqrt{27.2}$$

$$\boxed{x = 5.2}$$

4.



$$30^2 = 26^2 + 28^2 - 2(26)(28) \cos x$$

$$900 = 1460 - 1456 \cos x$$

$$-1460 \quad -1460$$

$$\frac{-560}{-1456} = \frac{-1456 \cos x}{-1456}$$

$$0.385 = \cos x \qquad \cos^{-1}(0.385)$$

$$\boxed{x = 1.701}$$

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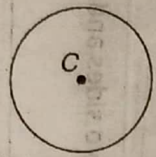
10.1 Notes (PART 1) – Circles and Circumference

G-C.A.1 – Recognize that all circles are similar

G-CO.A.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc

G-GMD.A.1 – Give an informal argument for the formulas for the circumference of a circle, and the volume and surface areas of a cylinder, cone, prism, and pyramid

A Circle is locus or set of all points in a plane equidistant from a given point called the center of the circle



Circle C or $\odot C$

Key Concept: Special Segments in a Circle

A radius (plural radii) is a segment with endpoints at the center and on the circle.

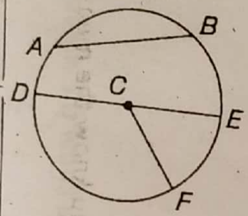
EX: $\overline{CD}$, $\overline{CF}$, $\overline{CE}$

A chord s segment with endpoints on the circle.

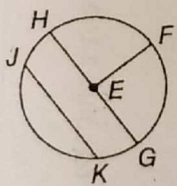
EX: $\overline{AB}$ and $\overline{DE}$

A diameter of circle is a chord that passes through the center and is made up of collinear radii

EX: DE



1./ Name the circle and identify each of the following (chord, diameter, radius)



circle E

Chord: JK

diameter: GH

radius: EF

Key Concept: Radius/Diameter relationships

If a circle has radius r and diameter d, the following relationships are true

Radius Formula $r=\frac{d}{2}$ or $r=\frac{1}{2}d$

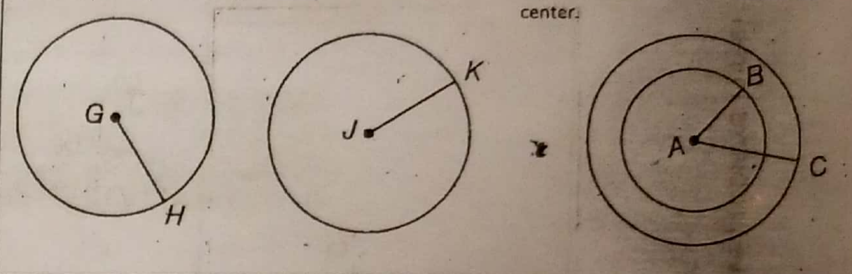
Diameter Formula $d=2r$

*If radius is 5, diameter is ____

Key Concept: Circle Pairs

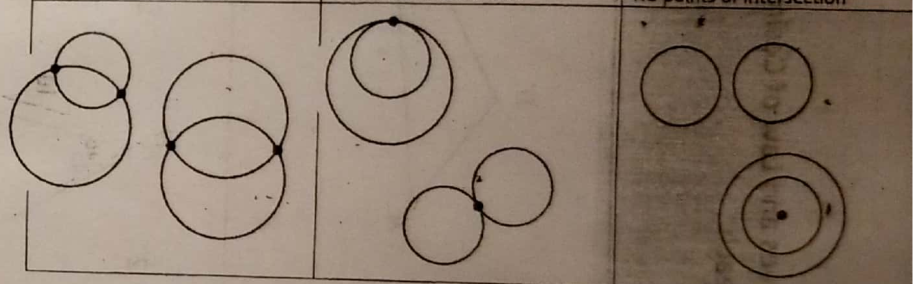
Two circles are congruent circles if and only if they have congruent radii.

Concentric Circles are coplanar circles that have the same center.

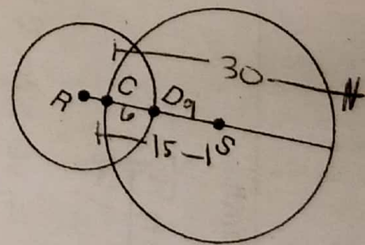


Two circles can intersect in two different ways

2 Points of Intersection	1 Point of Intersection	No points of Intersection



2. The diameter of circle S is 30 units, the diameter of Circle R is 20 units, and DS = 9 units. Find CD.



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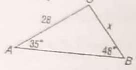
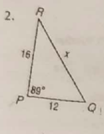
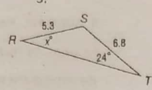
8.6 ASSI... ENT

Find x. Round angles to the nearest degree and side measures to the nearest tenth.

1./


2.


3.


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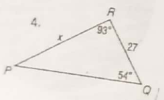

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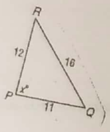

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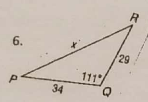

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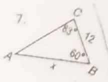

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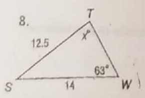

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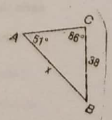

10.1 ASSIGNMENT (PART 1)

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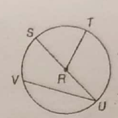


1./ a. Name the circle

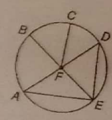
b./ Identify a chord that is also a diameter

c./ Is VU a radius? EXPLAIN

d./ If SU = 16.2 cm, What is T?

2. a. Identify a chord that is not a diameter.

b. If CF = 14 inches, what is the diameter of the circle?



c. Is AF ≅ EF? EXPLAIN

d./ If DA = 7.4 cm, what is EF?

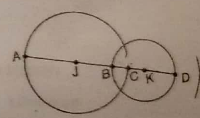
3. Circle J has a radius of 10 units, circle K has a radius of 8 units, and BC = 5.4 unit. Find each measure.

a. CK

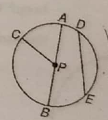
b. AB

c. JK

d. AD



4. a. Name the circle



b./ Name a radius

c./ Name a chord

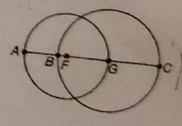
d./ Name a diameter

e./ Name a radius that is not part of the diameter

f./ Suppose the diameter is 16 cm. Find the radius

g./ If PC = 11 inches, find AB

5. The diameters of Circle F and Circle G are 5 and 6 units, respectively. Find each measure.



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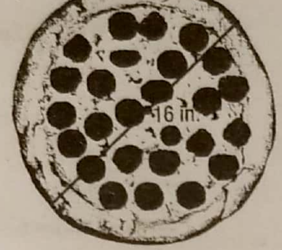
10.1 ASSIGNMENT (PART 2)

Name: Binder Copy

1./ RIDES. A circular ride at the fair has a diameter of 44 feet. What are the radius and circumference of the ride? Round to the nearest hundredth, if necessary.

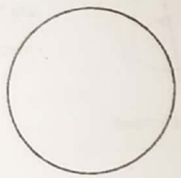
2. PIZZA. Find the radius and circumference of the pizza shown. Round to the nearest hundredth, if necessary.



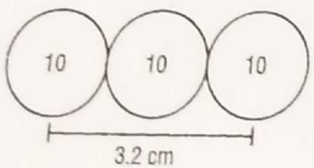
3. EXERCISE HOOPS. Taiga wants to make a circular loop that he can twirl around his body for exercise. He will use a tube that is 2.5 meters long.

a./ What will be the diameter of Taiga's exercise hoop? Round your answer to the nearest thousandth of a meter.

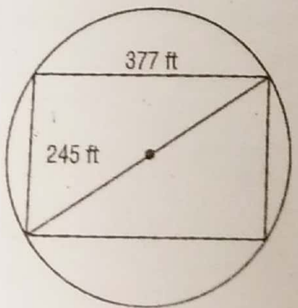
b. What will be the radius of Taiga's exercise hoop? Round your answer to the nearest thousandth of a meter.



4. COINS. Three identical circular coins are lined up in a row as shown. The distance between the centers of the first and third coins is 3.2 centimeters. What is the radius of one of these coins?



5. PLAZAS. A rectangular plaza has a surrounding circular fence. The diagonals of the rectangle pass from one point on the fence through the center of the circle to another point on the fence. Based on the information in the figure, what is the diameter of the fence?

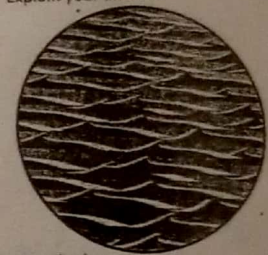


6. SUNDIALS. Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

a./ What is the radius of the sundial?

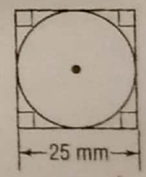
b./ Find the circumference of the sundial to the nearest hundredth.

7. CAKE CUTTING. Kathy slices through a circular cake. The cake has a diameter of 14 inches. The slice that Kathy made is straight and has a length of 11 inches.
Did Kathy cut along a RADIUS, a DIAMETER, or a CHORD of the circle? Explain your answer.

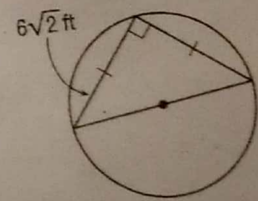


8. Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

a.



b.



HONORS must complete this, others can for extra credit.

9. PATIOS. Mr. Martinez is going to build the patio shown.

a./ What is the patio's approximate circumference?

b./ If Mr. Martinez changes the plans so that the inner circle has a circumference of approximately 25 feet, what should the radius of the circle be to the nearest foot?

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10.1 Notes (PART 2) – Circles and Circumference

G-C.A.1 – Recognize that all circles are similar

G-CO.A.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc

G-GMD.A.1 – Give an informal argument for the formulas for the circumference of a circle, and the volume and surface areas of a cylinder, cone, prism, and pyramid

Key Concept: Circumference
The Circumference of circle is the distance around the circle
To find the circumference: $C = \pi d$ or $C = 2\pi r$

1. Find each of the following using the given info

$d = 8\frac{1}{2}$ in, $r =$ 4.25, $C =$ 26.7

$C = 8.5\pi$

2. Find the diameter and radius of a circle to the nearest hundredth if the circumference of the circle is 106.4 mm

$C = \pi d$

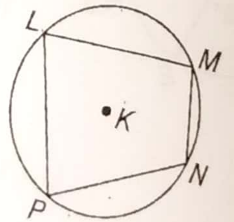
$\frac{106.4}{\pi} = \frac{\cancel{\pi} d}{\cancel{\pi}}$

$33.9 = d$

$16.95 = r$

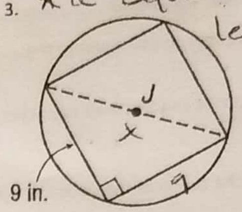
A polygon is inscribed in a circle if all of its vertices lie on the circle. A circle is circumscribed about a polygon if it contains all the vertices of the polygon

- Quadrilateral LMNP is inscribed in Circle K
- Circle K is circumscribed about Quad LMNP



Find the exact circumference of each circle by using the given inscribed or circumscribed polygon.

3. * a square with side length of 9



$C = \pi d$

$9^2 + 9^2 = x^2$

$162 = x^2$

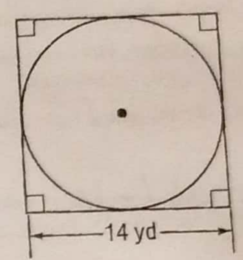
$9\sqrt{2} = x$

or use 45-45-90 rules

$C = \pi d$

$C = 9\sqrt{2}\,\pi$

4.



$C = 14\pi$

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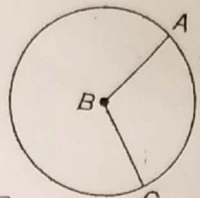
10.2 Notes (PART 1) – Measuring Angles and Arcs

G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

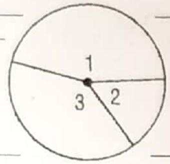
ESSENTIAL QUESTIONS: When lines intersect a circle, or within a circle, how do you measure the resulting angles, arcs, and segments?

OBJECTIVES: Identify central angles, major arcs, minor arcs, and semicircles and find their measures.

A Central Angle of a circle is an angle with a vertex in the center of the circle. Its sides contain radii of the circle. ∠ABC is the central angle.

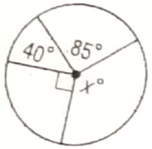


Key Concept: Sum of Central Angles
The sum of the measures of the central angles of a circle with no interior points in common is 360
Ex: ∠1 + ∠2 + ∠3 = 360



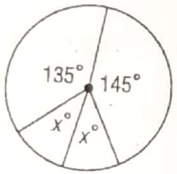
1./ Find x

a.



145

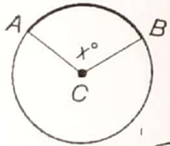
b.

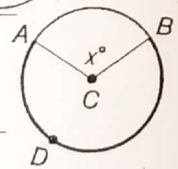


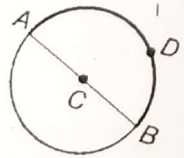
x=40

Arcs and Arc Measure

A minor arc is the shortest arc connecting two endpoints on a circle * 2 letters	The measure of a minor arc is less than 180 $m\overset{\frown}{AB} = m\angle ACB$
A major arc is the longest arc connecting two endpoints on a circle * 3 letters	The measure of a major arc is greater than 180 $m\overset{\frown}{ADB} = 360 - \overset{\frown}{AB}$
A semicircle is an arc with endpoints that lie on a diameter * 3 letters	The measure of a semicircle is 180 $m\overset{\frown}{ADB} = 180$





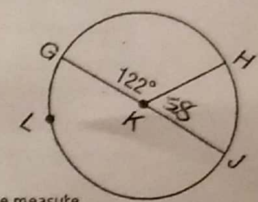


2. $\overline{GJ}$ is diameter of Circle K. Identify each arc as a MAJOR ARC, MINOR ARC, or SEMICIRCLE. Then find each measure.

a. $m\overset{\frown}{GH}$ 122 minor

b. $m\overset{\frown}{GLJ}$ 180 semicircle

c. $m\overset{\frown}{GLH}$ 360-122 238



Congruent arcs are arcs in the same or congruent circles that have the same measure

3. SPORTS. Refer to the circle graph to find each measure.

a. $m\overset{\frown}{CD}$

CD is volleyball which is 18% of circle

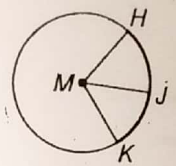
.18(360) = 64.8

b. $m\overset{\frown}{BC}$

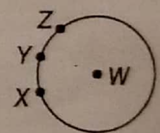
64.8

Adjacent Arcs are arcs in a circle that have exactly one point in common.

In Circle M, $\overset{\frown}{HJ}$ and $\overset{\frown}{JK}$ are adjacent arcs.



Postulate 10.1 Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.
EX:



4. Find each measure in Circle F.

a. $m\overset{\frown}{AED}$

63+90

=153

b. $m\overset{\frown}{ADB}$

360-117

243

c. $m\overset{\frown}{CE}$

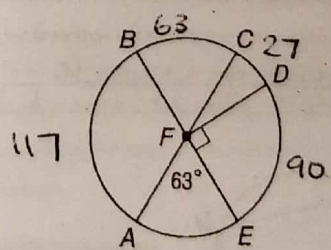
27+90

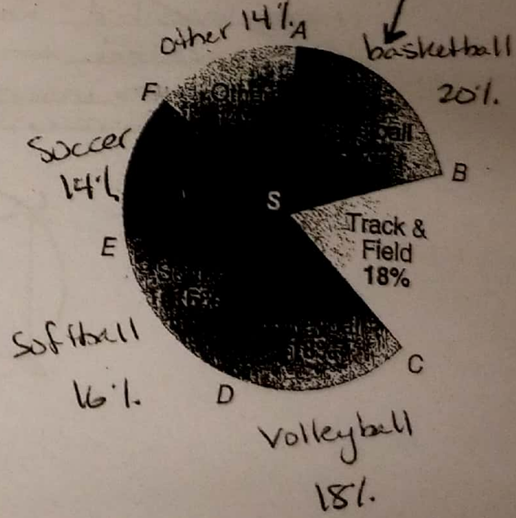
117

d. $m\overset{\frown}{ABD}$

117+63+27

207





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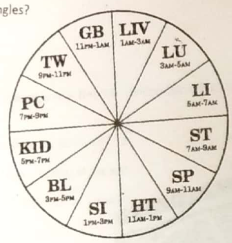
10.2 ASSIGNMENT (PART 1)

Name: Brooke Copy

1./ CLOCKS. Shiatsu is a Japanese massage technique. One of the beliefs is that various body functions are most active at various times during the day. To illustrate this, they use a Chinese clock that is based on a circle divided into 12 equal sections by radii.

What is the measure of any one of the 12 equal central angles?



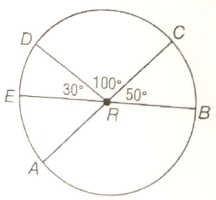
2. PIES. Yolanda has divided a circular apple pie into 4 slices by cutting the pie along 4 radii. The central angles of the 4 slices are $3x, 6x - 10, 4x + 10$ and $5x$ degrees. What exactly are the numerical measures of the central angles?

3. AC and EB are diameters of Circle R. Identify each arc as a MAJOR ARC, MINOR ARC or SEMICIRCLE. Then find its measure.

a./ $m\widehat{EA}$ b. $m\widehat{CB}$

c./ $m\widehat{DC}$ d. $m\widehat{DEB}$

e./ $m\widehat{AB}$ f. $m\widehat{CDA}$



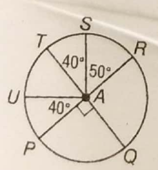
4. PR and QT are diameters of Circle A. Find each measure.

a. $m\widehat{UPQ}$ b. $m\widehat{PQR}$

c. $m\widehat{UTS}$ d. $m\widehat{RS}$

e. $m\widehat{RSU}$ f. $m\widehat{STP}$

g. $m\widehat{PQS}$ h. $m\widehat{PRU}$

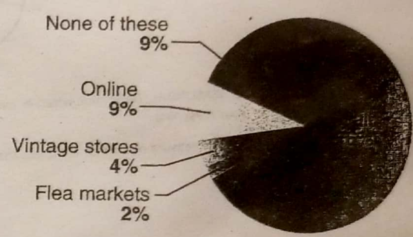


5. SHOPPING. The graph shows the results of a survey in which teens were asked where the best place was to shop for clothes.

a. What would be the arc measures associated with the mall and vintage stores categories?

b. Describe the kinds of arcs associated with the category "Mall" and the category "None of these.

c./ Are there any congruent arcs in this graph? EXPLAIN.



6. FOOD. The table shows the results of a survey in which Americans were asked how long food could be on the floor and still be safe to eat.

a. If you were to construct a circle graph of this information, what would be the arc measures associated with the first two categories?

b. Describe the kind of arcs associated with the first category and the last category

c./ Are there any congruent arcs in this graph? Explain

Dropped Food

Do you eat food dropped on the floor?

Not safe to eat	78%
Three-second rule*	10%
Five-second rule*	8%
Ten-second rule*	4%

Source: American Diabetic Association
* The length of time the food is on the floor.

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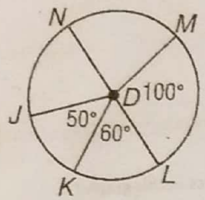
10.2 ASSIGNMENT (PART 2) Name: Binder Copy

Use Circle D to find the length of each arc. Round to the nearest hundredth

1./ $\widehat{LM}$ if the radius is 5 inches



2. $\widehat{MN}$ if the diameter is 3 yards

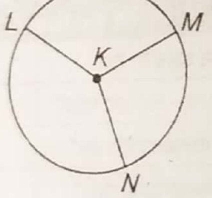
3. REASONING. Determine whether each statement is SOMETIMES, ALWAYS, or NEVER true. Explain your reasoning.

a. The measure of a minor arc is less than 180

b. If a central angle is obtuse, its corresponding arc is a major arc.

c./ The sum of the measures of adjacent arcs of a circle depends on the measure of the radius.

4. CHALLENGE. The measures of $\widehat{LM}$, $\widehat{MN}$, $\widehat{NL}$ are in the ratio 5:3:4. Find the measure of each arc.

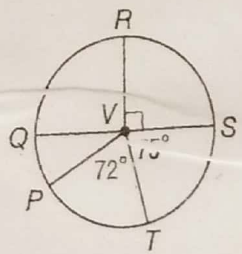


5. QS is a diameter of Circle V. Find each measure.

a. $m\widehat{STP}$

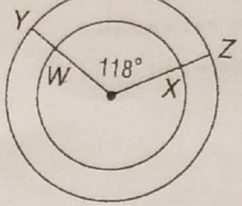
b. $m\widehat{QRT}$

c. $m\widehat{PQR}$



HONORS must complete these, extra credit for others.

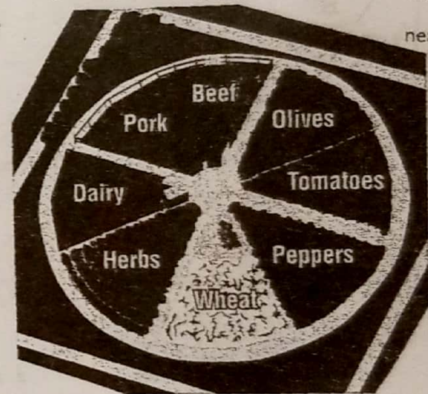
6. ERROR ANALYSIS. Brody says that $\widehat{WX}$ and $\widehat{YZ}$ are congruent since their central angles have the same measure. Selena says they are not congruent. Is either of them correct? Explain your reasoning.



7. FARMS. The *Pizza Farm* in Madera, California, is circle divided into eight equal slices. Each 'slice' is used for growing pizza ingredients.

a. What is the total arc measure of the slices containing olives, tomatoes, and peppers?

b. The circle is 125 feet in diameter. What is the arc length of one slice? Round to the nearest hundredth.

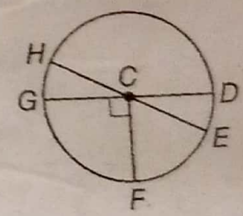


8. ALGEBRA. In Circle C, $m < HCG = 2x$ and $m < HCD = 6x + 28$. Find each measure

a. $m\widehat{EF}$

b. $m\widehat{HD}$

c. $m\widehat{HGF}$



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