10.2 Notes (PART 2) – Measuring Angles and Arcs

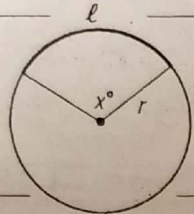
G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

ESSENTIAL QUESTIONS: When lines intersect a circle, or within a circle, how do you measure the resulting angles, arcs, and segments?

OBJECTIVES: Identify central angles, major arcs, minor arcs, and semicircles and find their measures.

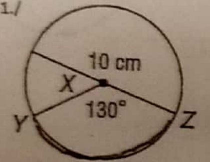
Arc Length is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of the circle, its length is a fraction of the circumference

Key Concept: Arc Length
The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360
Ex: arc length $= \frac{\text{degree}}{360} \cdot 2\pi r$



Find the length of $\widehat{ZY}$. Round to the nearest hundredth.

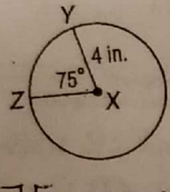
1./



$\frac{130}{360} \cdot 2\pi(5)$

≈ 11.34 cm

2.

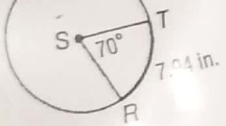


$\frac{75}{360} \cdot 2\pi(4)$

≈ 5.24 in

3. Find the measure. Round to the nearest hundredth.

circumference of circle S ______



arc length $= \frac{\text{degree}}{360} \cdot 2\pi r$

$7.94 = \frac{70}{360} \cdot 2\pi r$

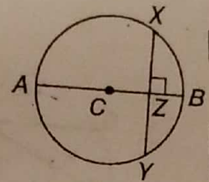
$7.94 = 1.22r$

$6.5 = r$

$C = 2\pi r$

$C = 2\pi(6.5)$

$\boxed{C = 40.8}$



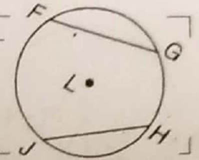
10.3 Notes – Arc and Chords

G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

ESSENTIAL QUESTIONS: What are the relationships between arcs, chords, and diameters?

OBJECTIVES: Recognize and use relationships between arcs and chords; Recognize and use relationships between arcs, chords, and diameters.

Theorem 10.2
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent
EX: $\overline{FG} \cong \overline{HJ}$ if and only if $\widehat{FG} \cong \widehat{HJ}$

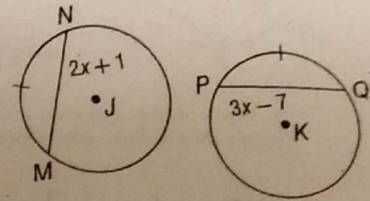


1./ In the figures, Circle J ≅ Circle K and $\widehat{MN} \cong \widehat{PQ}$. Find PQ

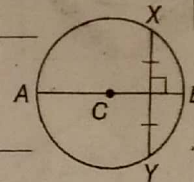
$3x - 7 = 2x + 1$

$x = 8$

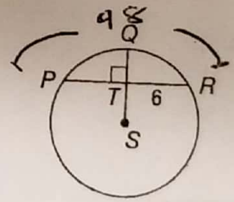
$PQ = 3(8) - 7 = 17$



10.3 If a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.
EX: $\overline{XZ} \cong \overline{ZY}$ and $\widehat{BX} \cong \widehat{BY}$
10.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.
EX:



2. In Circle S, $m\widehat{PQR} = 98$. Find $m\widehat{PQ}$

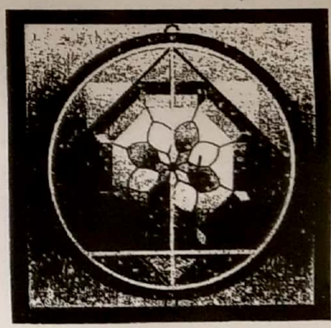


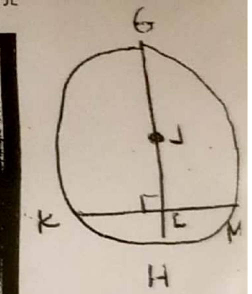
$\widehat{PR}$ is half of $\widehat{PQR}$,

so $m\widehat{PQ} = 49$

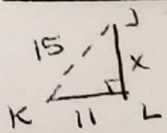
3. STAINED GLASS. In the stained glass window, diameter $\overline{GH}$ is 30 inches long and chord $\overline{KM}$ is 22 inches long. Find JL

30 radius is 15





* draw radius JK (dotted)



$a^2 + b^2 = c^2$

$x^2 + 11^2 = 15^2$

$x^2 + 121 = 225$

$x^2 = 104$

Binder Copy

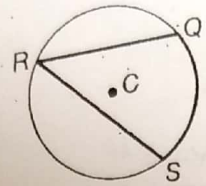
10.4 Notes (PART 1)– Inscribed Angles

G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

ESSENTIAL QUESTIONS: When lines intersect a circle, or within a circle, how do you find the measure of resulting angles, arcs, and segments?

OBJECTIVES: Identify and describe relationships involving inscribed angles; Prove properties of angles for a quadrilateral inscribed in a circle.

*Reminder: Central Angle: Vertex in center



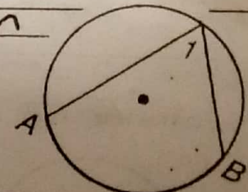
An inscribed angle has a vertex on a circle and sides that contain chords of the circle.

An intercepted arc has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle.

<QRS is an inscribed angle and $\overset{\frown}{QS}$ is its intercepted arc

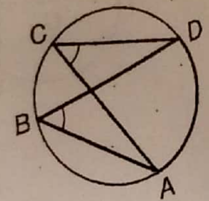
Theorem 10.6 Inscribed Angle Theorem
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

EX: $m\angle 1 = \frac{1}{2}\, m\overset{\frown}{AB}$



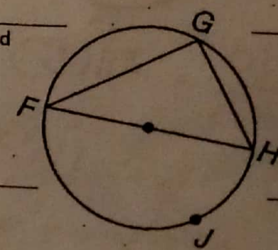
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the two angles are congruent.

EX: <C and <B both intercept $\overset{\frown}{AD}$
so <B ≅ <C



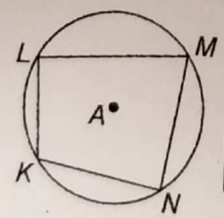
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.

EX: $\overset{\frown}{FJH}$ is a semicircle, then <G = 90



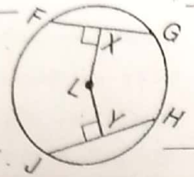
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary (add to 180)

EX: <K + <M = 180 , <L + <N = 180

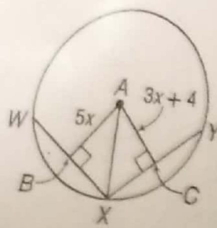


In the same circle or congruent circles, two chords are congruent if and only if they are equidistant from the center.

EX: $\overline{FG} \cong \overline{JH}$ if and only if $\overline{XL} \cong \overline{LY}$



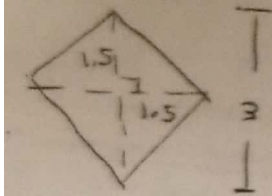
4. In Circle A, WX = XY = 22. Find AB.



$5x = 3x + 4$
$2x = 4$
$x = 2$
$AB = 5(2)$
$\boxed{AB = 10}$

5. DESIGN. Roberto is designing a logo for a friend's coffee shop according to the design, where each chord is equal in length. What is the measure of each arc and the length of each chord.

There are 4 arcs (each congruent), so $\frac{360}{4}$ each arc is 90°,



$a^2 + b^2 = c^2$
$1.5^2 + 1.5^2 = c^2$
$2.25 + 2.25 = c^2$
$4.5 = c^2$
$\sqrt{4.5} = c$
$\boxed{2.12 = \text{each chord}}$

Binder-Copy

10.4 Notes (PART 2)– Inscribed Angles

G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

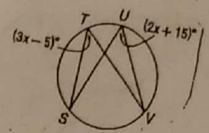
<u>ESSENTIAL QUESTIONS:</u> When lines intersect a circle, or within a circle, how do you find the measure of resulting angles, arcs, and segments?

<u>OBJECTIVES:</u> Identify and describe relationships involving inscribed angles; Prove properties of angles for a quadrilateral inscribed in a circle.

REMINDER: Central Angle

Inscribed Angle

1./ Find $m<T$



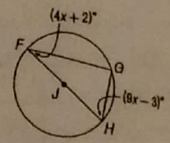
<T and <U both intercept $\overset{\frown}{SV}$, so $<T \cong <U$

$3x-5=2x+15$

$x=20$

$\boxed{m<T=55^\circ}$

2. Find $m<F$



<G is the inscribed angle for a semicircle, so <G = 90

$4x+2+9x-3+90=180$

$13x+89=180$

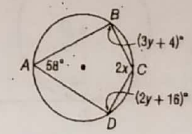
$13x=91$

$x=7$

$m<F=4(7)+2$

$\boxed{m<F=30}$

3. Find $m<C$ and $m<D$



opposite angles are supplementary

$2x+58=180$

$2x=122$

$\boxed{x=61}$

$m<C=2(61)$

$\boxed{m<C=122}$

$3y+4+2y+16=180$

$5y+20=180$

$5y=160$

$\boxed{y=32}$

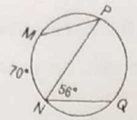
$m<D=2(32)+16$

$\boxed{m<D=80}$

1./ Find each measure. a. $m<P$ b. $m\overset{\frown}{PQ}$

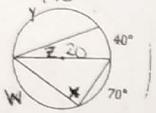
a. 35°

b. 112°



Find each measure.

2.



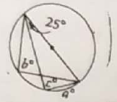
$z=20$

$y=140$

$x=90$

$w=110$

3.

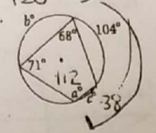


$a=50$

$b=90$

$c=90$

4.

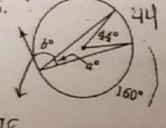


$a=112$

$c=38$

$b=120^\circ$

5.

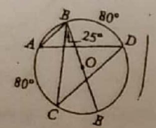


$156 \leftarrow 360-160-44$

$b=78$

$a=22$

6.



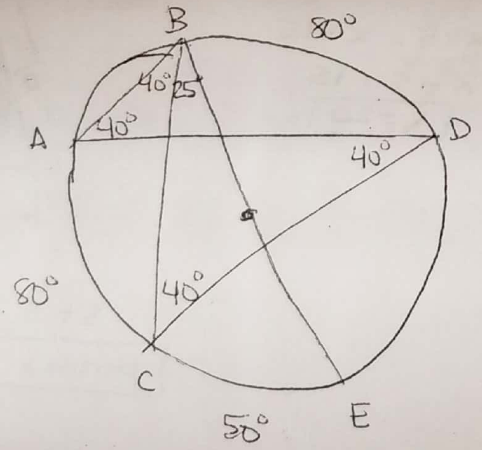
$m<A: 40^\circ$

$m\overset{\frown}{CE}: 50^\circ$

$m<C: 40^\circ$

$m<D: 40^\circ$

$m<ABE: 65^\circ$



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10.5 Notes – Tangents

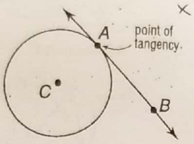
G-C.A.2 – Identify and describe relationships among inscribed angles, radii, and chords

G-C.A.3 – Construct the incenter and circumcenter of a triangle and use their properties to solve problems in context.

G-CO.D.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc)

ESSENTIAL QUESTIONS: How can the properties of circles, polygons, lines and angles be useful when solving geometric problems?

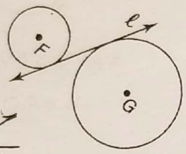
OBJECTIVES: Identify and describe relationships among tangents and radii; Identify and describe relationships among circumscribed angles and central angles; Construct the inscribed and circumscribed angles of a triangle. Construct a tangent line from a point.



*A tangent is a line in the same plane as a circle that intersects the circle in exactly one point called the point of tangency.

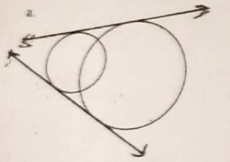
AB is a tangent and A is the point of tangency

*A common tangent is a line, ray, or segment that is tangent to two circles in the same plane.

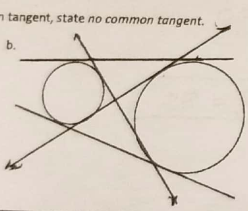


1. Draw the common tangents. If no common tangent, state *no common tangent*.

a.



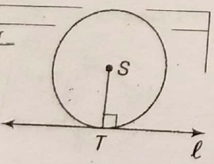
b.



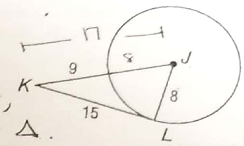
Theorem 10.10

In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

EX: A radius meeting a tangent forms a right angle



2. $\overline{JL}$ is a radius of Circle J. Determine whether $\overline{KL}$ is tangent to Circle J. Justify your answer.



$a^2+b^2=c^2$

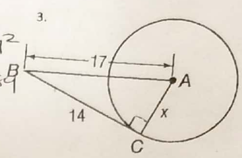
$8^2+15^2=17^2$

$289=289$ ✓

yes it is a tangent, since it's a right Δ.

Find the value of x. Assume that segments that appear to be tangent are tangent.

3.



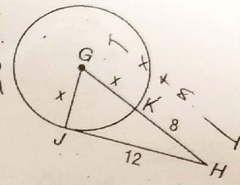
$x^2+14^2=17^2$

$x^2+196=289$

$x^2=93$

$x=9.6$

4.



$x^2+12^2=(x+8)^2$

$x^2+144=(x+8)(x+8)$

$x^2+144=x^2+16x+64$

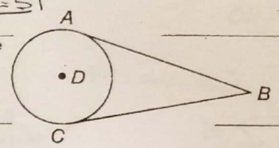
$144=16x+64$

$80=16x$

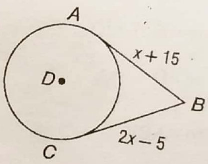
$\boxed{x=5}$

If two segments from the same exterior point are tangent to a circle, then they are congruent.

EX:



5. $\overline{AB}$ and $\overline{CB}$ are tangent to Circle D. Find the value of x.

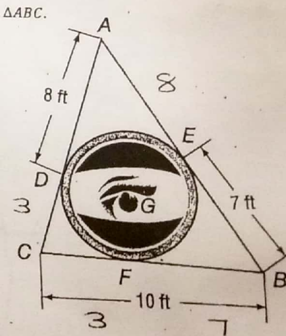


$2x-5=x+15$

$x-5=15$

$\boxed{x=20}$

6. GRAPHIC DESIGN. A graphic designer is giving directions to create a larger version of the triangular logo shown. If ΔABC is circumscribed about G, find the perimeter of ΔABC.



$8+8+3+3+7+7$

$\boxed{\text{perim}: 36\text{ ft}}$

Scanned by CamScanner