Guided Notes

Chapter 5
Polynomials and Polynomial Functions

Answer Key



**Unit Essential Questions**

What does the degree of a polynomial tell you about its related polynomial function?

For a polynomial function, how are factors, zeros and x-intercepts related?

For a polynomial equation, how are factors and roots related?

**Section 5.1: Polynomial Functions**

**Students will be able to classify polynomials**

**Students will be able to graph polynomial functions and describe end behavior**

**Warm Up**

 Simplify each expression by combining like terms.

1. 3*x* + 5*x* – 7*x*

*x*

1. –8*xy*2 – 2*x*2*y* + 5*x*2*y*

-8*xy*2 + 3*x*2*y*

1. –4*x* + 7*x*2 + *x*

7*x*2 – 3*x*

**Key Concepts**

Monomial – a real number, a variable, or a product of a real number and one or more variables with whole number exponents

Degree of a Monomial – the exponent of the variable

Polynomial – a monomial or the sum of monomials

Degree of a Polynomial – the largest degree of any term of the polynomial

Standard Form of a Polynomial – arranges the terms by degree in a descending numerical order P(x) = anxn + an-1xn-1 + ... + a1x + a0 where *n* is a nonnegative integer and an,….,a0 are real numbers.

 

**Examples**

1. Write each polynomial in standard form. Then classify it by degree and by number of terms.
2. 9 + *x*3

 *x*3 + 9; Cubic Binomial

1. 7*x*3 – 2*x*2 – 3*x*4

– 3*x*4 + 7*x*3 – 2*x*2 ; Quartic Trinomial

1. Write in standard form and classify by its degree and number of terms.
2. (*x*² - 3*x* + 4)(-5*x*² + 8*x* + 3)

-5x4 + 23x3 - 41x2 + 23x + 12; Quartic Polynomial of 5 terms

1. (4y² + 9y + 7) - (y² - 5y + 6)

3y2 + 14y + 1; Quadratic Trinomial

1. (x² + 4)(x + 2)²

x4 + 4x3 + 8x2 + 16x +16; Quartic Polynomial of 5 terms

**Key Concepts**

 Ending Behavior– direction of the graph to the far left and to the far right.



   

 Down and Up Up and Down Up and Up Down and Down

**Examples**

1. What is the ending behavior of the graph? Check using graphing calculator.
2. y = 4x3 – 3x

down and up

1. y = -2x4 +8x3 – 8x2 + 2

down and down

**Section 5.2 Part 1: Polynomials, Linear Factors, and Zeros**

**Students will be able to analyze the factored form of a polynomial**

**Students will be able to write a polynomial function from its zeros**

**Warm Up**

Factor each quadratic expression.

1. *x*2 + 7*x* + 12

(x + 3) ( x + 4)

1. *x*2 + 8*x* – *20*

(x + 10) (x – 2)

1. *x2* – 14*x* + 24

(x – 12) (x – 2)

**Examples**

1. Write 3*x*3 – 18*x*2 + 24*x* in factored form.

3x(x – 4)(x – 2)

**Key Concepts**

The following are equivalent statements about a real number *b* and a polynomial

P(x) = anxn + an-1xn-1 + ... + a1x + a0

* *x* – *b* is a *linear factor* of the polynomial P(*x*)
* *b* is a *zero* of the polynomial function y = f(*x*)
* *b* is a *root (or solution)* of the polynomial equation f(x) = 0
* *b* is an *x-intercept* of the graph y = f(x)

**Steps to Graphing a Polynomial Function**

1. Use the Zero-Product Property to find the zeros.
2. Find points between the zeros.
3. Determine ending behavior.
4. Use zeros, additional points, and ending behavior to sketch the graph.

**Examples**

1. Find the zeros of *y* = (*x* + 1)(*x* – 1)(*x* + 3). Then graph the function.

Zeros of the function are -3,-1, 1



**Key Concepts**

 **Factor Theorem**

The expression *x* – *a* is a factor of a if the value *a* is a zero of the related polynomial function.

**Examples**

1. Write a polynomial in standard form with zeros at 2, –3, and 0.

f(x) = x3 +x2 - 6x

**Section 5.2 Part 2: Polynomials, Linear Factors, and Zeros**

**Students will be able to analyze the factored form of a polynomial**

**Students will be able to identify relative maximums and minimums**

**Warm Up**

1. What are the zeros of f(x) = (x – 2)(x – 2)(x + 1)?

 2, 2, -1

1. Suppose someone listed the zeros as 2 and -1. Why might someone think this is a quadratic function?

 Answers vary

**Key Concepts**

Multiple Zero - repeated zero

Multiplicity - the number of times the zero occurs

**Example**

1. What are the zeros of ƒ(*x*) = *x*5 – 6*x*4 + 9*x*3?What are their multiplicities? How does the graph behave at these zeros?

Zeros are 0 and 3.

0 has a multiplicity of 3 and 3 has a multiplicity of 2.

The graph looks like a parabola at x = 3 and close to linear at x = 0

**Key Concepts**

 Relative maximum - the greatest y-value of the points in a region of graph

Relative minimum - the least y-value among nearby points on a graph

**Example**

1. What are the relative maximum and minimum of ƒ(*x*) = -4*x*3 + 12*x*2 + 4*x* – 12.

Relative Maximum: (2.15, 12.32)

Relative Minimum: (-0.15, -12.32)

**Section 5.3 Part 1: Solving Polynomial Equations**

**Students will be able to factor polynomial expressions**

**Warm Up**

Factor.

1. 6x2 + 12x

6x(x + 2)

1. 8x2 + 10x – 3

(4x – 1)(2x + 3)

1. x2 – 81

(x + 9)(x – 9)

**Key Concepts**

**Factoring by Grouping (4 terms)**

ax + ay + bx + by = a(x + y) + b(x + y) =(a + b)(x + y)

**Sum of Cubes**

a³ + b³ = (a + b)(a² - ab +b²)

**Difference of Cubes**

a³ – b³ = (a – b)(a² + ab + b²)

**Examples**

1. Factor *x*3 + 2*x*2 – 3*x* – 6.

 (x2 – 3)(x + 2)

1. Factor *x*3 – 8.

 (x – 2)(x2 + 2x + 4)

1. Factor x3 + 27.

 (x + 3)(x2 – 3x + 9)

**Section 5.3 Part 2: Solving Polynomial Equations**

**Students will be able to solve polynomial equations by factoring**

**Warm Up**

Factor.

1. x2 + 10x + 25

(x + 5)2

1. x3 + 2x2 – 4x – 8

(x + 2)2(x – 2)

1. x3 - 64

(x – 4)(x2 + 4x + 16)

**Key Concepts**

**Steps to solve a polynomial equation by factoring:**

1. Write the equation in the form P(x) = 0
2. Factor P(x)
3. Use the Zero-Product Property to find the roots

**Examples**

1. What are the real and imaginary solutions to 2x3 - 5x2 = 3x?

0, -1/2, 3

1. What are the real and imaginary solutions to x4 -3x2 = 4?

-2, 2, i, -i

1. What are the real and imaginary solutions to x3 = 1?

1, -½ + i√3/2, -½ - i√3/2

**Section 5.4 Part 1: Dividing Polynomials**

**Students will be able to divide polynomials using long division**

**Warm Up**

Divide using long division

1. 531 ÷ 3 2. 672 ÷ 21

177 32

**Key Concepts**

**The Divisor Algorithm**

You can divide polynomial P(x) by polynomial D(x) to get the quotient Q(x) and a remainder R(x).

If R(x) = 0, then D(x) and Q(x) are factors of P(x).

**Steps to Long Division of Polynomials**

1. Put each polynomial in standard form with zero coefficients where appropriate
2. Divide, multiply, subtract and repeat until the degree of the remainder is less than the divisor.

**Examples**

1. Divide *x*2 + 2*x* – 30 by *x* – 5.

 x + 7, R: 5

1. Determine whether *x* + 2 is a factor of the polynomial *x*2 + 10*x* + 16.

Yes, x + 2 is a factor of *x*2 + 10*x* + 16

**Section 5.4 Part 2: Dividing Polynomials**

**Students will be able to divide polynomials using synthetic division**

**Warm Up**

Evaluate for the given variables.



**12**

**Key Concepts**

 Synthetic Division – simplifies long division for dividing by a linear expression x – a.

**Steps for Synthetic Division**

1. Write coefficients (including zeros) of the polynomial in standard form. For the divisor, use the value for x such that x – a = 0 (use a).
2. Bring down the first coefficient.
3. Multiply the divisor and add it to the next coefficient.
4. Continue through the last coefficient.

**Example**

1. Use synthetic division to divide 5*x*3 – 6*x*2 + 4*x* – 1 by *x* - 3.

5x2 + 9x + 31, R 92

**Key Concepts**

**Remainder Theorem**

If you divide a polynomial P(x) by x – a, then the remainder is P(a).

**Example**

1. Given that P(x) = x5 – 3x4 – 28x3 + 5x + 20, what is P(-4)?

P(-4) = 0

**Section 5.5 Part 1: Theorems About Roots of Polynomial Equations**

**Students will be able to solve equations using the Rational Root Theorem**

**Warm Up**

 List all the factors of each number.

1. 12

 1, 2, 3, 4, 6, 12

1. 24

 1, 2, 3, 4, 6, 8, 12, 24

3. 44

 1, 2, 4, 11, 22, 44

**Key Concepts**

 **Rational Root Theorem**

Let P(x) = anxn + an-1xn-1 + ... + a1x + a0 be a polynomial with integer coefficients. Then there are a limited number of possible roots of P(x) = 0:

* Integer roots must be factors of a0.
* Rational roots must have reduced form p/q where p is an integer factor of ao and q is an integer factor of an.

**Steps to Finding Rational Roots**

1. List the possible rational roots using the Rational Root Theorem.
2. Test each possible rational root (REMINDER – you could also use synthetic division to evaluate).

**Examples**

1. Find the rational roots of 3*x*3 – *x*2 – 15*x* + 5 = 0.

1/3

1. Find the rational roots of 2*x*3 – *x*2 + 2*x* + 5 = 0.

-1

**Section 5.5 Part 2: Theorems About Roots of Polynomial Equations**

**Students will be able to use the conjugate root theorem and Decartes’ Rule of Signs**

**Warm Up**

Multiply.

1. (-4i)(6i)

24

1. (2 + i)(2 – i)

5

1. (1 + √3)(1 - √3)

-2

**Key Concepts**

**Conjugate Root Theorem**

If P(x) is a polynomial with rational coefficients, then the irrational roots of P(x) occur in pairs. If a - √b is an irrational root, then a + √b is also a root.

**Imaginary Root Theorem**

If the imaginary number a + bi is a root of a polynomial equation with real coefficients, then the conjugate a – bi also is a root.

**Examples**

1. A cubic polynomial P(x) has real coefficients. If 2 – 3i and ¾ are two roots of P(x), what is an additional root?

2 + 3i

1. A quartic polynomial P(x) has real coefficients. If √3 and 6 – 2i are two roots of P(x), what are the other two roots?

 6 + 2i and -√3

1. Find a third degree polynomial with rational coefficients that has roots –2, and 2 – *i*.

 P(x) = x3 – 2x2 – 3x + 10

**Key Concepts**

**Decartes’ Rule of Signs**

Let P(x) be a polynomial with real coefficients written in standard form.

* The number of positive real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(x) or is less than that by an even number.
* The number of negative real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(-x) or is less than that by an even number.

**Examples**

1. What does Decartes’ Rule of Signs tell you about the real roots of x3 - x2 + 1 = 0?

Since there are two sign changes, there are either 0 or 2 positive real roots.

Since there is one sign change, there is one negative real root.

**Section 5.6: The Fundamental Theorem of Algebra**

**Students will be able to use the Fundamental Theorem of Algebra to solve polynomial equations with complex solutions**

**Warm Up**

Solve x2 + 3x – 7 = 0.

 -3/2 + √37/2, -3/2 - √37/2

**Key Concepts**

 **The Fundamental Theorem of Algebra**

If P(x) is a polynomial of degree n ≥ 1, the P(x) = 0 has exactly n roots, including multiple and complex roots.

**Steps to find ALL roots of a polynomial equation**

1. Put polynomial in standard form and find possible rational roots.
2. Find a rational root.
3. Use synthetic division to find another factor.
4. Repeat until the quotient is factorable or a quadratic.
5. Solve.

**Examples**

1. Find the roots of 5*x*3 – 24*x*2 + 41*x* – 20 = 0.

2 + i, 2 – i, 4/5

1. Find the roots of x4 + *x*3 – 7*x*2 - 9*x* – 18 = 0.

3, -3, -1/2 + i√7/2, -1/2 - i√7/2